Proposal for C2Y
WG14 N3384

Title:	complex operators
Author, affiliation:	C FP group
Date:	2024-10-17
Proposal category:	Technical
Reference:	N3301

The suggested changes below move the complex operator definitions that are not particularly related to ISO/IEC 60559 out of Annex G and into the main body of the C standard.

The definitions for multiplicative operators (G.3.2 #1 and #2) and the definitions for additive operators (G.3.3 #1) support ISO/IEC 60559-consistent behavior for signed zeros and infinities, but the same definitions give the natural, most efficient implementation of the operators **+**, **-**, *****, and **/** with operands of mixed type domains (real and complex) for all implementations. Though the C23 usual arithmetic conversions allow implementations to use the formulas in these definitions, doing so is not currently required outside of Annex G. For example, the implementation of "complex op real" could convert the real operand to complex, with unexpected performance implications.

The current definitions in Annex G rely on tables that are not fully explained. The suggested changes are intended to be more explicit.

The suggested changes also introduce definitions for "complex ***** complex", "complex **/** complex", and "real **/** complex", referring to the usual mathematical formulas which the operators are intended to approximate.

In N3301, tables G1, G2, and G3 at no longer correct, presumably a result of editing to remove imaginary types. Details are not provided here because the suggested changes below remove the tables.

Note to editor: In the following changes, mathematical expressions are written entirely in ordinary font (Cambria), and C operators are written in a bold mono font (New Courier).

Suggested changes:

To 6.2.5 paragraph 17, append:

> … In this document complex values are sometimes written in the form $x + i\,y$, where x and y are the values of the real and imaginary parts, and i represents the mathematical imaginary unit, which has the property $i^2 = -1$. The form $x - i\,y$ is equivalent to $x + i\,(-y)$.

In 7.3.1 #3, delete the footnote:

…with the value of the imaginary unit;~~227)~~

~~227)The imaginary unit is a number *i* such that $i^2 = -1$.~~

In 6.5.6 under Semantics insert:

The following say the operator "is defined by" A + *i* B, where A and B indicate a real floating-point operation or value. This means the operator evaluates A and B to obtain the real and imaginary parts of the result. Thus, the result and any floating-point exceptions are completely specified.

— The ***** operator where one operand is real with value x and the other operand is complex with value $u + i\,v$ is defined by $(x * u) + i\,(x * v)$.

— The **/** operator where the first operand is complex with value $x + i\,y$ and the second operand is real with value u is defined by $(x / u) + i\,(y / u)$.

The following say the operator "computes" A + *i* B, where A and B are mathematical expressions. This means the real and imaginary parts of the result approximate A and B respectively. The formulas given by A and B do not indicate how the results are to be evaluated, nor do they necessarily give desirable results when infinities or NaNs are involved (see Annex G). The computation should avoid undue overflow and underflow (see 7.3.4).

— The ***** operator where both operands are complex with values $x + i\,y$ and $u + i\,v$ computes $(xu - yv) + i\,(yu + xv)$. The operator should be commutative.

— The **/** operator where both operands are complex with values $x + i\,y$ and $u + i\,v$ computes $(xu + yv)/(u^2 + v^2) + i\,(yu - xv)/(u^2 + v^2)$.

— The **/** operator where the first operand is real with value x and the second operand is complex with value $u + i\,v$ computes $(xu)/(u^2 + v^2) + i\,(-xv)/(u^2 + v^2)$.

In 6.5.7 under Semantics insert:

The following say the operator "is defined by" A + *i* B, where A and B indicate a real floating-point operation or value. This means the operator evaluates A and B to obtain the real and imaginary parts of the result. Thus, the result and any floating-point exceptions are completely specified.

— The **+** operator where both operands are complex with values $x + i\,y$ and $u + \mathrm{i}\,v$ is defined by $(x + u) + i\,(y + v)$.

— The **+** operator where one operand is real with value x and the other operand is complex with value $u + i\,v$ is defined by $(x + u) + i\,v$.

— The − operator where the first operand is complex with value $x + i\, y$ and the second operand is complex with value $u + i\, v$ is defined by $(x - u) + i\, (y - v)$.

— The − operator where the first operand is real with value x and the second operand is complex with value $u + i\, v$, the difference is defined by $(x - u) + i\, (-v)$.

— The − operator where the first operand is complex with value $x + \mathrm{i}\, y$ and the second operand is real with value u, the difference is defined by $(x - u) + i\, y$.

In G.3.2 delete paragraphs 1 and 2.

Delete G.3.3.