

# Atomic maximum/minimum

## Proposal to extend atomic with maximum/minimum operations

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Audience: LEWG; SG1 - Concurrency

### Abstract

Add integer 'max' and 'min' operations to the set of operations supported in `<atomic>`. There are minor adjustments to function naming necessitated by the fact that 'max' and 'min' do not exist as infix operators. Also, in contrast to the existing atomic operations (and previous versions of this proposal), it is unspecified whether a write occurs if the new value is the same as the old value.

### Revision history

- P0493R0 (2016-11-08): Original proposal
- P0493R1 (2020-05-08): Add motivation for defining new atomics as read-modify-write. Clarify status of proposal for new-value-returning operations. Align with C++17.
- P0493R2 (2021-05-11): Change proposal to make the store unspecified if the value does not change. Align C++20.

### Introduction

This proposal extends the atomic operations library to add atomic maximum/minimum operations. These were originally proposed for C++ in N3696 as particular cases of a general "priority update" mechanism, which atomically combined reading an object's value, computing a new value and conditionally writing this value if it differs from the old value. In contrast to N3696, we propose atomic maximum/minimum operations where it is unspecified whether or not the store takes place if the new value happens to be the same as the old value. A future proposal may reintroduce the concept of a conditionalized atomic update.

This paper benefited from discussion with Mario Torrecillas Rodriguez, Nigel Stephens and Nick Maclaren, and updates have benefited from discussion in the SG1 Concurrency group.

### Background and motivation

Atomic addition (fetch-and-add) was introduced in the NYU Ultracomputer [Gottlieb 1982], has been implemented in a variety of hardware architectures, and has been standardized in C and C++. Atomic maximum/minimum operations (fetch-and-max) have a history almost as long as atomic addition, e.g. see [Lipovski 1988], and have also been implemented in various hardware architectures but are not currently standard in C and C++. This proposal fills the gap.

Atomic maximum/minimum operations are useful in a variety of situations in multithreaded applications:

- optimal implementation of lock-free shared data structures - as in the motivating example later in this paper
- reductions in data-parallel applications: for example, OpenMP (<https://computing.llnl.gov/tutorials/openMP/#REDUCTION>) supports maximum/minimum as a reduction operation
- recording the maximum so far reached in an optimization process, to allow unproductive threads to terminate
- collecting statistics, such as the largest item of input encountered by any worker thread.

Atomic maximum/minimum operations already exist in several other programming environments, including OpenCL (<https://www.khronos.org/registry/cl/specs/opencl-2.0-opencl.pdf>), and in some hardware implementations. Application need, and availability, motivate providing these operations in C++.

The proposed language changes add atomic max/min to `<atomic>`, with some syntactic adjustment due to the fact that C++ has no infix operators for max/min, and with a slight difference in semantics as described below.

The existing atomic operations (e.g. `fetch_and`) have the effect of a read-modify-write, irrespective of whether the value changes. This is how atomic max/min are defined in several APIs (OpenCL, CUDA, C++AMP, HCC) and in several hardware architectures (ARM, RISC-V). However, some hardware (POWER) implements atomic max/min as an atomic read-and-conditional-store. For performance and portability, to allow efficient implementation on a variety of architectures, this proposal leaves it unspecified whether the store happens if the new value is the same as the old value. That is, the proposed C++ atomic max/min functions can be implemented using atomic max/min hardware operations where the store always happens in such a situation, where it never happens, or where the hardware itself leaves it unspecified.

Hardware which implements these operations as read-and-conditional-write may offer a performance advantage at the cost of possible correctness, where the following instruction makes an assumption as to the write half-barrier (i.e. "release" semantics) of the preceding "min" or "max" operation. Similarly, platforms where these operations are read-modify-always-write may sacrifice performance where there is no actual change in value, but make it easier to write correct code, where the following instruction can take for granted the "release" semantics of the preceding "min" or "max" operation. Portable and correct code may have to introduce an additional write barrier in case the value did not change, but such a barrier might penalize performance on platforms where these operations are guaranteed read-modify-write since the write will have implemented the requested semantics. This could be made easier by an addition of a predefined macro which would be always set to e.g. 1 on platforms where an additional section like this is needed, e.g. `__cpp_lib_atomic_maxmin_always_writes` (and 0 otherwise). **We ask for feedback** as to whether or not such macro is needed.

### Summary of proposed additions to `<atomic>`

The current `<atomic>` provides atomic operations in several ways:

- as a named non-member function template e.g. `atomic_fetch_add()` returning the old value

- as a named member function template e.g. `atomic<T>::fetch_add()` returning the old value
- as an overloaded compound operator e.g. `atomic<T>::operator+=()` returning the new value

Adding 'max' and 'min' versions of the named functions is straightforward. Unlike the existing atomics, max/min operations exist in signed and unsigned flavors. The atomic type determines the operation. There is precedent for this in C, where all compound assignments on atomic variables are defined to be atomic, including sign-sensitive operations such as divide and right-shift.

The overloaded operator `atomic<T>::operator op=(n)` is defined to return the new value of the atomic object. This does not correspond directly to a named function. For max and min, we have no infix operators to overload. So if we want a function that returns the new value we would need to provide it as a named function. However, for all operators the new value can be obtained as `fetch_op(n) op n`, (the standard defines the compound operator overloads this way) while the reverse is not true for non-invertible operators like 'and' or 'max'. Thus the functions would add no significant functionality other than providing one-to-one equivalents to `<atomic>`'s existing compound operator overloads. Following some of the early literature on atomic operations ([Kruskal 1988] citing [Draughon 1967]), we suggest that if required, names should have the form `replace_op`. The current revision of this paper demonstrates what the `replace_min` and `replace_max` functions would look like; **we ask for feedback** whether or not these should be removed; in case these are to be left in, **we ask for feedback** whether or not to add additional overloads taking memory ordering parameter. We must stress that any inconsistency with the existing atomic functions is based on the lack of infix representation of these operations in the language syntax, rather than because of any difference in the nature of the operations in the language execution model.

This paper proposes operations on integral and pointer types only. If both this proposal and floating-point atomics as proposed in P0020 are adopted then we propose that atomic floating-point maximum/minimum operations also be defined, in the obvious way.

## References

- [Almasi]: "Highly Parallel Computing" 2nd ed., George S. Almasi and Allan Gottlieb,
- [Draughon 1967]: "Programming Considerations for Parallel Computers", E.R. Draughon et al., Courant Inst, 1967
- [Gong 1990]: "A Library of Concurrent Objects and Their Proofs of Correctness", Chun Gong and Jeanette M. Wing, 1990, <http://www.cs.cmu.edu/~wing/publications/CMU-CS-90-151.pdf>
- [Gottlieb 1982]: "The NYU Ultracomputer - Designing an MIMD Shared Memory Parallel Computer", Gottlieb et al., ICCA, 1982
- [Kruskal 1988]: "Efficient Synchronization on Multiprocessors with Shared Memory", Clyde P. Kruskal et al., Ultracomputer Note #105, 1988
- [Lipovski 1988]: "A Fetch-And-Op Implementation for Parallel Computers", G.J. Lipovski and Paul Vaughan, 1988

## Changes to the C++ standard

The following text outlines the proposed changes, based on N4868 (DIS 14882:2020).

### 31: Atomic operations library [atomics]

#### 31.2: Header `<atomic>` synopsis [atomics.syn]

```
namespace std {
    // 31.9, non-member functions
    ...
    template<class T>
        T atomic_fetch_max(volatile atomic<T>*, typename atomic<T>::value_type) noexcept;
    template<class T>
        T atomic_fetch_max(atomic<T>*, typename atomic<T>::value_type) noexcept;
    template<class T>
        T atomic_fetch_max_explicit(volatile atomic<T>*, typename atomic<T>::value_type, memory_order) noexcept;
    template<class T>
        T atomic_fetch_max_explicit(atomic<T>*, typename atomic<T>::value_type, memory_order) noexcept;
    template<class T>
        T atomic_fetch_min(volatile atomic<T>*, typename atomic<T>::value_type) noexcept;
    template<class T>
        T atomic_fetch_min(atomic<T>*, typename atomic<T>::value_type) noexcept;
    template<class T>
        T atomic_fetch_min_explicit(volatile atomic<T>*, typename atomic<T>::value_type, memory_order) noexcept;
    template<class T>
        T atomic_fetch_min_explicit(atomic<T>*, typename atomic<T>::value_type, memory_order) noexcept;
    ...
}
```

#### 31.7.3: Specializations for integral types [atomics.ref.int]

```
namespace std {
    template <> struct atomic_ref<integral> {
        ...
        integral fetch_max(integral, memory_order = memory_order_seq_cst) const noexcept;
        integral fetch_min(integral, memory_order = memory_order_seq_cst) const noexcept;
        ...
        integral replace_max(integral) const noexcept;
        integral replace_min(integral) const noexcept;
    };
}
```

Change:

Effects: ... These operations are atomic read-modify-write operations (6.9.2.2)

to

Effects: ...Except for `fetch_max` and `fetch_min`, these operations are atomic read-modify-write operations (6.9.2.2). For `fetch_max` and `fetch_min`, if the new value is the same as the existing value, it is unspecified whether or not the new value is written; if it is written, the effect is of a read-modify-write operation.

Change:

Remarks: For signed integer types, the result is as if the object value and parameters were converted to their corresponding unsigned types, the computation performed on those types, and the result converted back to the signed type.

to

Remarks: Except for `fetch_max` and `fetch_min`, for signed integer types, the result is as if the object value and parameters were converted to their corresponding unsigned types, the computation performed on those types, and the result converted back to the signed type. For `fetch_max` and `fetch_min`, the computation is performed according to the integral type of the atomic object.

Add the following text:

```
integral A::replace_key(integral operand) const noexcept;
```

Requires: These operations are only defined for keys 'max' and 'min'.

Effects: `A::fetch_key(operand)`

Returns: `std::key(A::fetch_key(operand), operand)`

After

```
integral operatorop=(integral operand) const noexcept;
```

add "These operations are not defined for keys 'max' and 'min'."

### 31.7.5: Partial specialization for pointers [atomics.ref.pointer]

```
namespace std {
  template <class T> struct atomic_ref<T *> {
    ...
    T* fetch_max(T *, memory_order = memory_order::seq_cst) const noexcept;
    T* fetch_min(T *, memory_order = memory_order::seq_cst) const noexcept;
  };
}
```

Change

Effects: ... These operations are atomic read-modify-write operations (6.9.2.2)

to

Effects: ... Except for `fetch_max` and `fetch_min`, these operations are atomic read-modify-write operations (6.9.2.2). For `fetch_max` and `fetch_min`, if the new value is the same as the existing value, it is unspecified whether or not the new value is written; if it is written, the effect is of a read-modify-write operation.

Add the following text:

```
T* A::replace_key(T* operand) const noexcept;
```

Requires: These operations are only defined for keys 'max' and 'min'.

Effects: `A::fetch_key(operand)`

Returns: `std::key(A::fetch_key(operand), operand)`

### 31.8.3: Specializations for integers [atomics.types.int]

```
namespace std {
  template <> struct atomic<integral> {
    ...
    integral fetch_max(integral, memory_order = memory_order_seq_cst) volatile noexcept;
    integral fetch_max(integral, memory_order = memory_order_seq_cst) noexcept;
    integral fetch_min(integral, memory_order = memory_order_seq_cst) volatile noexcept;
    integral fetch_min(integral, memory_order = memory_order_seq_cst) noexcept;
    ...
  };
}
```

In table 144, [tab:atomic.types.int.comp], add the following entries:

Key	Op	Computation
max		maximum as computed by <code>std::max</code> from <code>&lt;algorithm&gt;</code>
min		minimum as computed by <code>std::min</code> from <code>&lt;algorithm&gt;</code>

Change:

Effects: ... These operations are atomic read-modify-write operations (6.9.2.2)

to

Effects: ... Except for `fetch_max` and `fetch_min`, these operations are atomic read-modify-write operations (6.9.2.2). For `fetch_max` and `fetch_min`, if the new value is the same as the existing value, it is unspecified whether or not the new value is written; if it is written, the effect is of a read-modify-write operation.

Add the following text:

```
C A::replace_key(M operand) volatile noexcept;  
C A::replace_key(M operand) noexcept;
```

Requires: These operations are only defined for keys 'max' and 'min'.

Effects: `A::fetch_key(operand)`

Returns: `std::key(A::fetch_key(operand), operand)`

After

```
T* operatorop=(T operand) noexcept;  
add "These operations are not defined for keys 'max' and 'min'."
```

### 31.8.5: Partial specialization for pointers[`atomic.types.pointer`]

```
namespace std {  
    template <class T> struct atomic<T*> {  
        ...  
        T* fetch_max(T*, memory_order = memory_order_seq_cst) volatile noexcept;  
        T* fetch_max(T*, memory_order = memory_order_seq_cst) noexcept;  
        T* fetch_min(T*, memory_order = memory_order_seq_cst) volatile noexcept;  
        T* fetch_min(T*, memory_order = memory_order_seq_cst) noexcept;  
        ...  
    };  
}
```

In table 145, [`tab:atomic.types.pointer.comp`], add the following entries:

Key	Op	Computation
max		maximum as computed by <code>std::max</code> from <code>&lt;algorithm&gt;</code>
min		minimum as computed by <code>std::min</code> from <code>&lt;algorithm&gt;</code>

Change:

Effects: ... These operations are atomic read-modify-write operations (6.9.2)

to

Effects: ... These operations are atomic read-modify-write operations, except that for the 'max' and 'min' operations, if the new value is the same as the existing value, it is unspecified whether or not the new value is written.

Add:

```
C A::replace_key(M operand) volatile noexcept;  
C A::replace_key(M operand) noexcept;
```

Requires: These operations are only defined for keys 'max' and 'min'.

Effects: `A::fetch_key(operand)`

Returns: `std::key(A::fetch_key(operand), operand)`

After

```
T* operatorop=(T operand) noexcept;  
add "These operations are not defined for keys 'max' and 'min'."
```

## Motivating example

Atomic fetch-and-max can be used to implement a lockfree bounded queue, as explained in [Gong]:

```

typedef struct {
    elt item; /* a queue element */
    int tag; /* its generation number */
} entry;

typedef struct rep {
    entry elts[SIZE]; /* a bounded array */
    int back;
} reptime;

reptime queue;

void Enq(elt x) {
    int i;
    entry e, *olde;
    e.item = x; /* set the new elements item to x */
    i = READ(&(queue.back)) + 1; /* get a slot in the array for the new element */
    while (true) {
        e.tag = i / SIZE; /* set the new elements generation number */
        olde = EXCHANGE(&(queue.elts[i % SIZE]), -1, &e);
        /* exchange the new element with slots
        value if that slot has not been used */
        if (olde->tag == -1) { /* if exchange is successful */
            break; /* get out of the loop */
        }
        ++i; /* otherwise, try the next slot */
    }
    FETCH_AND_MAX(&(queue.back), i); /* reset the value of back */
}

elt Deg() {
    entry e, *olde;
    int i, range;
    e.tag = -1; /* make e an empty entry */
    e.item = NULL;
    while (true) { /* keep trying until an element is found */
        range = READ(&(queue.back)) - 1; /* search up to back-1 slots */
        for (i = 0; i <= range; i++) {
            olde = EXCHANGE(&(queue.elts[i % SIZE]), i / SIZE, &e);
            /* check slot to see if it contains the oldest element */
            if (olde->tag != -1) { /* if so */
                return(olde->item); /* return the item in it */
            }
        } /* otherwise try the next one */
    }
}

```

A similar C++ example was used in the original version of this paper, due to Bronek Kozicki.

A queue class can be used as follows:

```

int main()
{
    queue<int> q(16);
    assert(q.post(42));
    int d;
    assert(q.read(d));
    assert(d == 42);
    assert(not q.read(d));
}

```

A naive implementation of the queue follows:

```

#include <atomic>
#include <utility>
#include <cstdint>

template <typename T>
class queue
{
    // Rounded up logarithm with base of 2
    static int log2(int s)
    {
        --s;
        int r = 0;
        while (s)
        {
            s >>= 1;
            r += 1;
        };
        return r;
    }
}

```

```

// Actual data storage, contains queued value and data stamp for this slot
struct slot
{
    slot() : value(), stamp(0)
    { }

    T value;
    std::atomic_long stamp;
};

public:
queue(const queue&) = delete;
queue& operator=(const queue&) = delete;

explicit queue(int s) : head_(0) , tail_(0) , bits_(log2(s)) , size_(1 << bits_) , buffer_(nullptr)
{
    buffer_ = new slot[size_];
}

~queue()
{
    // Must not be called when either post() or read() are running in other
    // threads. Such calls must be completed before destruction
    delete[] buffer_;
}

bool post(T&& v) noexcept(true)
{
    slot* ptr = nullptr;    // Store the data to here
    long expected = 0;      // compared against ptr->stamp
    unsigned long head = head_.load();
    for (;;)
    {
        ptr = &buffer_[index(head)];
        expected = stamp(head);
        const long newstamp = expected + 1;
        const long oldstamp = ptr->stamp.load();
        if (oldstamp == expected)
        {
            const unsigned long next = head + 1ul;
            // Try to claim ownership of the slot
            if (head_.compare_exchange_weak(head, next))
            {
                ptr->stamp = newstamp;
                break;
            }
            // else head has been updated
        }
        else if (oldstamp > expected)
            head = head_.load(); // claimed by another thread already
        else
            return false; // overflowing, i.e. ptr is to be read yet
    }

    ptr->value = std::move(v);
    ptr->stamp = expected + 2;
    return true;
}

bool read(T& v) noexcept(true)
{
    slot* ptr = nullptr;    // Read the data from here
    long expected = 0;      // compared against ptr->stamp
    unsigned long tail = tail_.load();
    for (;;)
    {
        // Optimize for case when data needs to be read, but check that
        // there is actually anything in there.
        if (tail == head_.load())
            break; // Must not advance tail beyond head

        ptr = &buffer_[index(tail)];
        expected = stamp(tail) + 2;
        const long newstamp = expected + 1; // = stamp(tail) + 3
        const long oldstamp = ptr->stamp.load();
        if (oldstamp == expected)
        {
            const unsigned long next = tail + 1ul;
            // Try to claim ownership of the slot
            if (tail_.compare_exchange_weak(tail, next))
            {
                ptr->stamp = newstamp;
                break;
            }
            // else tail has been updated
        }
        else
    }
}

```

```

        tail = tail_.load(); // claimed by another thread already
    }
    ptr = nullptr;
}
if (ptr)
{
    v = std::move(ptr->value);
    ptr->stamp = expected + 2;
    return true;
}
return false;
}
private:
// Calculate head/tail position inside buffer_ array
constexpr int index(unsigned long h) const
{
    return (h & (size_ - 1ul));
}
// Calculate lap number for high bits in slot->stamp
constexpr long stamp(unsigned long h) const
{
    return (h & ~(size_ - 1ul)) >> (bits_ - 2);
}
std::atomic_ulong          head_; // slot being written
std::atomic_ulong          tail_; // slot being read
const int                  bits_; // = log2(size_)
const int                  size_; // must be power of 2
slot*                       buffer_;
};

```

This version suffers from a performance problem, because `read()` will not be able to skip over the slot still-being-written to following it slots which are ready for read. The following improved version uses `atomic_fetch_max`:

```

#include <atomic>
#include <utility>
#include <cstdint>

template <typename T>
class queue
{
    // Rounded up logarithm with base of 2
    static int log2(int s)
    {
        --s;
        int r = 0;
        while (s)
        {
            s >>= 1;
            r += 1;
        };
        return r;
    }

    // Actual data storage, contains queued value and data stamp for this slot
    struct slot
    {
        slot() : value(), stamp(0)
        { }

        T value;
        std::atomic_ulong stamp;
    };

public:
    queue(const queue&) = delete;
    queue& operator=(const queue&) = delete;

    explicit queue(int s) : head_(0) , tail_(0) , bits_(log2(s)) , size_(1 << bits_) , buffer_(nullptr)
    {
        buffer_ = new slot[size_];
    }

    ~queue()
    {
        // Must not be called when either post() or read() are running in other
        // threads. Such calls must be completed before destruction
        delete[] buffer_;
    }

    bool post(T&& v) noexcept(true)
    {
        slot* ptr = nullptr; // Store the data to here
    }

```

```

long expected = 0; // CAS against ptr->stamp
unsigned long head = head_.load();
unsigned long next = 0; // Next value of head
for (;;)
{
    next = head + 1ul;
    ptr = &buffer_[index(head)];
    expected = stamp(head);
    const long newstamp = expected + 1;
    // Not going to revisit this slot in next iteration, so "strong" is required
    if (ptr->stamp.compare_exchange_strong(expected, newstamp))
        break;

    // Advance to next slot if this was claimed by another thread
    if (expected >= newstamp)
        head = next;
    else
        return false; // overflowing, i.e. ptr is to be read yet
}
atomic_fetch_max(head_, next);

ptr->value = std::move(v);
ptr->stamp = expected + 2;
return true;
}

bool read(T& v) noexcept(true)
{
    slot* ptr = nullptr; // Read the data from here
    long expected = 0; // CAS against ptr->stamp
    unsigned long tail = tail_.load();
    unsigned long next = tail; // Next value of tail
    for (;;)
    {
        // Optimize for case when data needs to be read, but check that
        // there is actually anything in there.
        const unsigned long head = head_.load();
        if (tail == head)
            break; // Must not advance tail beyond head

        ptr = &buffer_[index(tail)];
        expected = stamp(tail) + 2;
        const long newstamp = expected + 1; // = stamp(tail) + 3
        // Not going to revisit this slot in next iteration, so "strong" is required
        if (ptr->stamp.compare_exchange_strong(expected, newstamp))
        {
            // Advance tail if no slot was being written
            if (next == tail)
                next = tail + 1ul;
            break;
        }

        ptr = nullptr;
        // Advance tail if no slot was being written.
        if (expected >= newstamp && next == tail)
            next = tail + 1ul;
        tail += 1ul;
    }
    atomic_fetch_max(tail_, next);

    if (ptr)
    {
        v = std::move(ptr->value);
        ptr->stamp = expected + 2;
        return true;
    }

    return false;
}

private:
// Calculate head/tail position inside buffer_ array
constexpr int index(unsigned long h) const
{
    return (h & (size_ - 1ul));
}

// Calculate lap number for high bits in slot->stamp
constexpr long stamp(unsigned long h) const
{
    return (h & ~(size_ - 1ul)) >> (bits_ - 2);
}

std::atomic_ulong head_; // slot being written
std::atomic_ulong tail_; // slot being read
const int bits_; // = log2(size_)
const int size_; // must be power of 2

```

```
}; slot*
```

```
buffer_;
```