Abstract

In Toronto (July 2017), EWG merged the Concepts TS into C++20, but without the introducer and terse syntaxes and without constrained variables and non-type parameters due to concerns. This paper attempts a design that:

- builds directly from the terse and introducer syntax in the Concepts TS and Stroustrup’s P0694R0;
- tries to address the concerns about the Concepts TS terse and introducer syntaxes, especially that function templates be visually distinct and that deduction of the same concept default to independent-type; and
- leaves the option of getting the Concepts TS syntaxes in the future via simple defaults (e.g., omit empty `{}`).

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1 Overview

1.1 Background and motivation

Unifying generic and ordinary programming using the terse (a.k.a. abbreviated, natural, etc.) constraint syntax has been a key goal since concepts were first presented to the committee. The original 2003 concepts paper N1536 (Stroustrup, Dos Reis) included a section on this syntax, and ever since the concepts designers have expressed the view that concepts was about more than just constraints.

Although draft C++0x concepts and later N3351 did not mention the terse syntax, the concepts designers accepted C++0x concepts as a point-in-time interim step knowing that the terse syntax could be layered on to get to “full concepts.” Given that we are now some 10 years beyond C++0x concepts, and have been working on concepts within WG21 for 14 years (and longer before WG21), there is a greater sense of urgency to finally get “full concepts” including the unification of generic and ordinary programming with the terse syntax, and not decouple the terse syntax for long.

The Toronto meeting Tuesday evening session had over 70 experts present, and conducted two main polls:

- There was a strong consensus to merge the Concepts working paper (as amended by other changes approved earlier in the day) into the C++20 working draft, but excluding “for now” the introducer and terse syntaxes along with constrained variables and constrained non-type template parameters.
- There was overwhelming support to continue to pursue a terse syntax that could overcome the concerns with the Concepts TS terse syntax, especially that function templates be visually distinct and that deduction of the same concept default to independent-type semantics.

This paper presents a design for a terse syntax for applying concepts that:

- builds directly from the terse and introducer syntax in the Concepts TS and Stroustrup’s P0694R0 (embraces the TS introducer syntax and allows it generally throughout the language, not just in one place);
- attempts to address the concerns about the Concepts TS terse and introducer syntaxes, especially that function templates be visually distinct and that deduction of the same concept default to independent-type semantics; and
- deliberately leaves the door open to getting exactly the Concepts TS terse parameter, introducer, and constrained variable and non-type parameter syntaxes and semantics (except only for the TS’s same-type semantics default for parameters) in the future by adding a simple default if the committee is comfortable doing so once we gain further experience with the feature.

In particular, the difference between the Concepts TS and this paper’s proposal is often “just {}”, as in:

<table>
<thead>
<tr>
<th>Concepts TS and P0694R0</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>void sort(Sortable&amp; s);</td>
<td>void sort(Sortable{}&amp; s);</td>
</tr>
</tbody>
</table>

See §4 for many side-by-side examples, including all of the main P0694R0 examples.

This paper builds on previous papers including the original Stroustrup and Dos Reis N1536 (2003) concepts proposal which did include the terse syntax, the Concepts TS (2015), Ballo and Sutton’s N3878 (Jan 2014), Smith and Dennett’s P0587 (Feb 2017), Vandevoorde’s [Van17] (Mar 2017 EWG presentation), Van Eerd and Ballo’s P0464R2 (Mar 2017), Brown’s N4434 (Apr 2015), Stroustrup’s thorough treatments in P0694R0 and P0695R0 (Jun 2017), Honermann’s P0696R0 (June 2017), and Spicer, Tong, and Vandevoorde’s P0691R0 (June 2017).
1.2 Design principles

**Note** These principles apply to all design efforts and aren’t specific to this paper. Please steal and reuse.

The primary design goal is conceptual integrity [Brooks 1975], which means that the design is coherent and reliably does what the user expects it to do. Conceptual integrity’s major supporting principles are:

- **Be consistent**: Don’t make similar things different, including in spelling, behavior, or capability. Don’t make different things appear similar when they have different behavior or capability.—For example, this paper follows the principle in the Concepts TS and Stroustrup’s [P0694R0] that generic programming and ordinary programming should be consistent, especially that writing the boilerplate text “template<>” should not be needed for many generic algorithms, and that **auto** be the least constrained concept. It diverges from the Concepts TS where there are three different “short” constraint syntaxes (besides explicit **requires**). Also, this paper’s proposed constrained type introducer can be used to constrain any deduced type (after deduction), including a type parameter, auto variable, or auto return type.

- **Be orthogonal**: Avoid arbitrary coupling. Let features be used freely in combination.—For example, in this paper we follow the principle that applying a constraint should not change the meaning of code; applying a constraint should (only) reject a type that does not match that constraint, not change semantics in any other way. This principle is essential to preserving that **auto** is in fact the least constrained concept (otherwise replacing **auto** with a more-constrained concept would have other semantic effects than just increasing the constraint), and avoids adding surprises such as that merely adding or changing a constraint (changing **auto** to a concept name, or changing one concept name to another) could sometimes make a function be a template and sometimes not, sometimes make a two-parameter heterogenous function be usable only homogeneously, sometimes make a return type not be deduced from the return expression, and so on. All of these would be sources of user surprise, add contextual meaning that is hostile to refactoring, and have the usual other negative consequences of language inconsistencies.

- **Be general**: Don’t restrict what is inherent. Don’t arbitrarily restrict a complete set of uses. Avoid special cases and partial features.—For example, this paper suggests that since the Concepts TS allows single-type concept constraints in template<> parameter lists, we should allow all constraints there, including multi-type concept constraints, as was previously suggested in [N3878] (Ballo and Sutton). This paper also proposes a general way to introduce constrained type names “on the fly” at point of use.

These also help satisfy the principles of least surprise and of including only what is essential, and result in features that are additive and so directly minimize concept count (and therefore also redundancy and clutter).

1.3 Acknowledgments

Thanks for the following for their feedback, including all authors of recent papers in this area: Botond Ballo, Walter Brown, Chandler Carruth, Casey Carter, James Dennett, Gabriel Dos Reis, Hal Finkel, Tom Honermann, Erich Keane, Robert Klarer, Thomas Köppe, Adam Martin, Eric Niebler, Arthur O’Dwyer, Jakob Riedle, Barry Revzin, Richard Smith, John Spicer, Bjarne Stroustrup, Andrew Sutton, Daveed Vandevoorde, Tony Van Eerd, and Ville Voutilainen.
2 Proposal overview

2.1 Concept{T1, T2} introduces constrained type names

Today, a concept name followed by a list of type arguments in <> checks the constraint:

\[ \text{ConceptName} \ < \ \text{identifier-list} \ < \ ] \] // checks one or more already-existing type names

Note This paper neither proposes nor precludes extensions to this syntax except as noted in §2.2.3.

This paper proposes a constrained type introducer where a type concept name followed by a list of type names in {} introduces the names, as in the Concepts TS introducer syntax (see §5.2 for the alternative of using []):

\[ \text{ConceptName} \ { \text{identifier-list}_{\text{opt}} } \] // introduces zero or more constrained type names

which can be used to introduce constrained type names “on the fly” within existing grammar elements (notably, in template parameter lists and in declarations of parameters, return types, and local variables).

Future simplification → Concepts TS terse syntax via a default

In the future, we could add: “where if there are no introduced names we can also omit {}”.

It is deliberate that if we added that, we would have the current Concepts TS terse syntax. My aim is to construct a syntax that works well now and also gives us a path to adopting the Concepts TS terse syntax in the future, if the committee gains comfort with viewing empty {} as unnecessary once we gain experience with the feature.

Notes This proposal is for concepts whose parameters are all types.

If you see an introducer Concept{T}, you know T is introducing a name for a type that is being deduced and then constrained to Concept.

This proposal is not currently proposing auto{/*name*/} to be able to give a name to the type of a template non-type parameter, a parameter of template type parameter type, or an ordinary variable of deduced type. This could be proposed in the future as a compatible extension, and would be useful in cases like [](auto{T}&& x) { return f(std::forward<T>(x)); }.

This proposal is not currently proposing nesting of Concept1{Concept2{/...*/}}. This could be proposed in the future as a compatible extension if it is found useful, such as to mean introducing a type name that satisfies both constraints.

This proposal is not currently proposing introducing variadic names (e.g., Concept{T...}). This could be proposed in the future as a compatible extension if it is found useful.

Introducing constrained type names has the following semantics:

- Multiple introductions of the same type name introduce the same type, whether it is constrained by the same concept or different concepts. It is the same name, therefore must be the same type. The type must satisfy all the constraints in which it is mentioned.
- Introducing different type names introduces independent types, whether they are constrained by the same concept or different concepts. They are different names, therefore need not be the same type.
- Name(s) are not required; introduce a name only if it’s needed again (such as to declare another variable of the same type). When omitted, the semantics are that the compiler invents an unspecified unique name, as with other unnamed entities (e.g., lambdas). Therefore it follows that introducing anonymous unique [therefore different] type names introduces independent types (see §5.1.5).
2.2 Illustrative examples of `Concept{...}`

2.2.1 In template declarations and definitions

For class templates, variable templates, and alias templates, here is how to use the syntax to introduce constrained type names in the template parameter list:

```cpp
template<typename T>
class matrix { vector<T> v; /*...*/ }; // use the type name
// means: template<class T> requires Number<T> /*etc.*/
```

```cpp
template<typename N>
constexpr N pi = N{3.1415926535897932385L}; // use the type name
// means: template<class N> requires FloatingNumber<N> /*etc.*/
```

```cpp
template<typename N>
using T = something<N>; // use the type name
// means: template<class N> requires Number<N> /*etc.*/
```

For function templates, we can introduce constrained type names in the template parameter list:

```cpp
template<typename Mergeable<In1, In2, Out>> Out merge(In1, In1, In2, In2, Out); // use the type name
// means: template<class In1, class In2, class Out> requires Mergeable<In1, In2, Out> /*etc.*/
```

and in a parameter or return type position (if the name is unneeded and omitted, the language as-if invents one):

```cpp
void f(Number<N> n, Sortable<S>& s) {
    N t = n + something(); // use the type name
    Value_type<S> val = *s.begin(); // use the type name
}
// means: template<class N, class S> requires (Number<N> && Sortable<S>) /*etc.*/
```

```cpp
Input_iterator<I> find(I, I, Equality_comparable<Value_type<I>>{}); // use the type name
// means: template<class I> requires Input_iterator<I> /*etc.*/
```

```cpp
auto multiply(Number<N>, N) -> Number{} // constrain the return type (in trailing return)
    { /*... use the type name ... */
    // means: template<class N, class _T> requires Number<N> && Number<_T> /*etc.*/

void sort(Sortable{}&);
// means: template<class _T> requires Sortable<_T> /*etc.*/
```

If the same type name is introduced more than once in the scope of a single template declaration or definition, the type must meet all the constraints. For example:

```cpp
void f(Number<N>, Number<N>); // same except conversions allowed on second param
// means: template<class N> requires Number<N> /*etc.*/
```

and in the following `N` must satisfy two constraints:

```cpp
void g(Number<N>, Addable<N>);
// means: template<class N> requires (Number<N> && Addable<N>) /*etc.*/
```
Type parameters and non-type parameters are made unambiguous by consistently requiring {} when introducing a type name, including in constrained type parameters (where the Concepts TS and C++20 working draft currently support the syntax without {}; this proposal reserves that syntax for non-type parameters). For example:

```cpp
template<class N, auto a_value> void f(); // unconstrained
template<Number{N}, Number{} a_value> void f(); // constrained
```

\*^\* with these {} required around a type name, this is unambiguous

Notes
This does not preclude removing empty {} in the future, which is still an unambiguous syntax (note that the parameter name must be required):

```cpp
template<Number{TypeName}, Number Value> void f(); // if empty {} were optional
```

Also, it avoids the current visual ambiguity where the constrained template would be written:

```cpp
template<typename T> concept Number = /*...*/; // type concept
template<int T> concept Numeric = /*...*/; // value concept
```

where it is not possible to see that one is a type parameter and one is a value parameter without consulting the definitions of the two concepts. This paper proposes a general constraint introduction syntax for type concepts only, and the removal of the ambiguous syntax `template<Number TypeName, Numeric Value> void f();`. For a discussion of value concepts and paths where those lead, see §5.4.

2.2.2 In function (or function template) bodies (variables)
For example, here is how to use the syntax to constrain the type of a variable:

```cpp
Number{N} n = 42;   // n is an int, and N is int
Number{} n = 42;    // same, except not introducing the name N
```

2.2.3 Using <> and {} together: Bound arguments via currying

Note This subsection is an independent detail on which nothing else depends.

In this proposal, using `Concept<>` with N arguments binds the first N parameters of `Concept`. For example, `ComparableTo<X>Y>` is equivalent to `ComparableTo<X,Y>`.

In this proposal, using `Concept{}` with N introduced names binds the first N parameters of `Concept`. For example, `ComparableTo{X}Y>` is equivalent to `ComparableTo{X,Y}`.

When the end of a series of <> and {} is reached, the total number of all arguments and introduced names must match or exceed the number of non-defaulted parameters of `Concept`.

This lets us mix <> and {} more freely and consistently. For example, given this concept:

```cpp
template <class T, class U>
concept ComparableTo = requires(T const& t, U const& u)
{ { t == u } -> bool; { u == t } -> bool; };
```

we can express introducing two names T and U that satisfy `ComparableTo` like this:

```cpp
template <ComparableTo{T,U}>   // means: template <class T, class U> requires ComparableTo<T,U>
```

and distinctly express introducing a name that is `ComparableTo` an existing type name X by using both <> and {}:
template <ComparableTo<T<X>> // means: template <class T> requires ComparableTo<T,X>

template <ComparableTo<X>U>> // means: template <class U> requires ComparableTo<X,U>

**Notes** Some of this syntax, such as A<B>{C}, just falls out of the existing grammar with the proposed {} extension, since A<B> is a concept name and we’re just applying the {} syntax to that as in the base case.

When combining <> and {}, to express the constraint

```cpp
template<class T> requires ComparableTo<T,X> // direct <> order: T X
```

in the TS and the C++20 working paper we currently would write this as:

```cpp
template<ComparableTo<X>{U}> // means: template <class U> requires ComparableTo<X,U>
```

but this proposal suggests making the order consistent while we have the chance, before casting the current syntactic inversion in stone in the standard, and instead write this as:

```cpp
template<ComparableTo{T<X>>> // proposed order: T X
```

which makes ComparableTo<T,X> and Comparable{T<X> consistent.

---

### 2.2.4 Overview comparison with Concepts TS (see §4 for detailed examples)

The Concepts TS has three “short” constraint syntaxes (besides requires) that are all different with different restrictions, shown below. The only one that is fully general is the introducer syntax in its special grammar position. This paper proposes eliminating the special grammar position, but embracing the introducer syntax and applying it generally and directly within the current grammar to constrain the places where we already write typename or auto to introduce a type.

This supports strictly more uses of a non-requires syntax (it appears to eliminate the need to resort to requires), with consistent syntax and no novel grammar position. For example, it avoids the Concepts TS and P0694R0 limitation that you can have multiple constraints (in the template<> parameter list, #1 below), or you can have a multi-type constraint (in the special introducer syntax, #3 below), but you cannot have both (without resorting to low-level requires which is undesirable), which violate both consistency and generality.

<table>
<thead>
<tr>
<th>Concepts TS and P0694R0</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>// extended syntax 1: no multi-type constraints</td>
<td>// extended syntax 1: multi-type constraints ok</td>
</tr>
<tr>
<td>template&lt;/<em>Mergeable ???</em>/&gt; void merge(In1, In1, In2, In2, Out);</td>
<td>template&lt;Mergeable{In1, In1, Out}&gt; void merge(In1, In1, In2, In2, Out);</td>
</tr>
<tr>
<td>// extended syntax 2: no ability to name the type</td>
<td>// extended syntax 2: optionally naming type ok</td>
</tr>
<tr>
<td>void sort(Sortable&amp; s) { // using S = remove_reference_t&lt;decltype(s)&gt;;} }</td>
<td>void sort(Sortable{S}&amp; s) { }</td>
</tr>
<tr>
<td>// novel syntax 3: only one constraint allowed</td>
<td>// n/a, use 1 or 2 above</td>
</tr>
<tr>
<td>Sortable{S} /<em>Number{N}</em>/ void sort(S&amp; s, N pivot);</td>
<td></td>
</tr>
<tr>
<td>// (TS only) syntax 1 ambiguous and inconsistent // for constrained non-type parameters</td>
<td>// introduced type names are always inside {} // nontype/parm/var names are always outside {}</td>
</tr>
<tr>
<td>template&lt;Number Type, Number Value&gt; void f(Number parameter) { Number variable; }</td>
<td>template&lt;Number{Type}, Number{Value} value&gt; void f(Number{} parameter) { Number{} variable; }</td>
</tr>
</tbody>
</table>
3 Proposal

3.1 Part 1: Allow all constraints in template parameter lists

3.1.1 Proposal

For template parameter lists, allow all constraints (including multi-type constraints) where the semantics for type parameters are identical to today's class or typename, and for non-type parameters are identical to today's auto, except that additionally a deduced type can be constrained and (optionally) be named.

In a template parameter list (including of a template template parameter or generic lambda), permit the constrained type introducer `Concept{Ti/*, ..., Tn*/}` as an entry in the list. The following rules apply:

- The list of introduced names must be nonempty ($n>0$).
- The semantics are the same as:
  - replace `Concept{Ti/*, ..., Tn*/}` with `typename Ti/*, ..., typename Tn*/` for each Ti that does not already appear as a template type parameter in the same list (possibly via another constrained type introducer); and
  - add (or extend) the requires-clause with `requires Concept<Ti/*, ..., Tn*/>`. 

In a template parameter list (including of a template template parameter or generic lambda), permit the constrained type introducer `Concept{/*/}` as the type of a non-type (value) parameter in the list. The following rules apply:

- The list of introduced names must have 0 or 1 entries ($n<=1$).
- The name of the parameter is required.
- If the list is empty, the compiler invents an unspecified unique name for T.
- The semantics are the same as:
  - replace `Concept{T}` with `auto`;
  - make T an alias for the deduced type of the non-type parameter (if that name appears elsewhere in the list, the deduction must agree with the other uses); and
  - add (or extend) the requires-clause with `requires Concept<T>`. 

For example, to illustrate the main cases, writing this template parameter list:

```cpp
template<
    Sortable{S},
    Mergeable{In1, In2, Out},
    template<Iterator{I}> class X,
    Number{N} MaxSize
>
```

is equivalent to writing this:

```cpp
template<
    typename S,
    typename In1, typename In2, typename Out,
    template<typename I> requires Iterator<I> class X,
    auto MaxSize /*, typename N = decltype(MaxSize)*/ /* exposition only */
>
```
requires
    Sortable<S> &&
    Mergeable<In1, In2, Out> &&
    Number<N>

And writing this:

    template<
        Mergeable<In1, In2, Out>,
        RandomAccessIterator<Out>,
        Number<T> MaxSize
    >

is equivalent to writing this:

    template<
        typename In1, typename In2, typename Out,
        auto MaxSize /*, typename _T = decltype(MaxSize)*/ /* exposition only */
    >

    requires
        Mergeable<In1, In2, Out> &&
        RandomAccessIterator<Out> &&
        Number<_T>

Writing this:

    template<class Out, Mergeable<In1, In2, Out>>
    void f(In1, In2, Out);

is equivalent to writing this:

    template<class Out, class In1, class In2>
    requires Mergeable<In1, In2, Out>
    void f(In1, In2, Out);

Similarly for generic lambdas, writing this:

    auto f = []<Sortable<S>, Mergeable<In1, In2, Out>> (S, In1, In2, Out) { };

is equivalent to writing this:

    auto f = []<typename S, typename In1, typename In2, typename Out>
    requires Sortable<S> && Mergeable<In1, In2, Out>
    (S, In1, In2, Out) { };

3.1.2 Discussion
The Concepts TS and the current working paper allow single-type constraints in the template<> list as follows:

    template<Sortable S> // same meaning in Concepts TS and C++20 WP
    void sort(S&);

    // means: template<class S> requires Sortable<S> /*etc.*/

However, allowing only single-type constraints is an arbitrary restriction that violates the principle of generality.
Also, the lack of any decoration creates a visual ambiguity with non-type parameters; for example,

```cpp
template<Number N> void f();  // different meanings in Concepts TS and C++20 WP
// means this: template<typename N> requires Number<N> void f();
// or this: template<auto N> requires Number<decltype(N)> void f();
```

where the TS assigns the first meaning if `Number` takes a type parameter, and the second if it takes a non-type parameter, in addition to the ordinary non-type parameter meaning if `Number` happens to be a type rather than a concept. This is a three-way visual ambiguity at the call site. Furthermore, in the second meaning it does not permit a direct way to name the type (the user must resort to a later `decltype` as today). This proposal avoids these ambiguities and inconsistencies.

Like N3878, this paper proposes that the `template<>` parameter list allow all constraints including multi-type constraints using the same syntax as the Concepts TS introducer syntax, which requires `{}`. For example:

```cpp
template< Mergeable{In1, In2, Out}> // proposed (not allowed in Concepts TS)
Out merge(In1, In1, In2, In2, Out);
// means: template<class In1, class In2, class Out> requires Mergeable<In1, In2, Out> //etc.*/
```

**Future simplification → Concepts TS introducer syntax via a default**

In the future, we could add: “where if there is only one item in <> we can also omit `template<>`”.

It is deliberate that if we added that, we would have the current Concepts TS introducer syntax. Because the EWG Toronto polls were to progress concepts without the introducer and terse syntaxes, and continue to work on the terse syntax, I am focusing on the terse syntax again. But this design deliberately aims to be able to achieve both introducer and terse syntaxes as they appear in the Concepts TS if we want them in the future, and as special cases of a natural generalization (reducing from the four distinct syntaxes allowed by the Concepts TS).

Template template parameters are handled recursively:

```cpp
template< template< Mergeable{In1, In2, Out}> class X >
Out do_merge(X);
// means: template< template<class In1, class In2, class Out> requires Mergeable<In1, In2, Out> class X > /etc.*/
```

The proposed generalization encourages users to write better code by removing reasons to resort to `requires`-clauses and needless concepts proliferation. As described in N3878, the Concepts TS currently has no syntax for expressing a constrained template that has a multi-type constraint and another constraint, without resorting to a low-level `requires` clause, or inventing a one-off function-specific named concept as an ugly workaround that essentially nobody will actually do. For example, we should be able to write:

```cpp
// 1: illegal in Concepts TS, proposed herein
template <Mergeable{In1, In2, Out}, SortableIterator{Out}>
void merge_then_sort(In1 first1, In1 last1, In2 first2, In2 last2, Out out);
```

but this cannot be expressed today without forcing the user to resort to a `requires`-clause (which I argue is lower-level and less disciplined; it also makes conjunctions explicit which opens the door for the user to as easily write a disjunction and I’d rather remove that temptation):

```cpp
// 2: drop to using requires-clauses (lower-level; Concepts TS style with requires)
template <typename In1, typename In2, typename Out>
```
requires Mergeable<In1, In2, Out> & SortableIterator<Out>
void merge_then_sort(In1 first1, In1 last1, In2 first2, In2 last2, Out out);

or alternatively resort to writing an algorithm-specific concept (which I argue is undesirable and leads to verbosity and name littering) and using introducer syntax:

// 3: proliferate one-off concepts (Concepts TS style with introducer syntax)
template <typename In1, typename In2, typename Out>
concept MergeableAndOutIsSortable
= Mergeable<In1, In2, Out> & SortableIterator<Out>;

void merge_then_sort(In1 first1, In1 last1, In2 first2, In2 last2, Out out);

3.2 Part 2: Constrained type names in parameter/return values

3.2.1 Proposal
For parameter and return values, the semantics are identical to today’s template type parameters and auto return types except that additionally a deduced type can be constrained and (optionally) be named. Adding a constraint to an auto return type does not affect whether or not the return type is deduced from return statements, and does not affect whether or not the function is a template.

Replacement for a constrained return type is performed before replacement for a constrained parameter. This ensures that the choice of leading or trailing return type syntax does not change the semantics of the function template, in particular to allow deduction on constrained parameter types whether the constrained return type is declared lexically first or last (see example below).

In the declaration of a parameter type for a function template (which could be a lambda) or a template argument of such a type (e.g., vector<Number{N}>), permit the constrained type introducer Concept{/*T*/} as the type. The following rules apply:

- The list of introduced names must have 0 or 1 entries (n<=1).
- If the list is empty, the compiler invents an unspecified unique name for T.
- The semantics are the same as moving the constraint into the template parameter list:
  - replace Concept{T} with T; and
  - add (or extend) the function template’s template parameter list with Concept{T} (see §3.1).

In the declaration of a trailing return type for a function or function template (which could be a lambda), permit the constrained type introducer Concept{} as the type. The following rules apply:

- The list of introduced names must be empty (n==0).
- The compiler invents an unspecified unique name for T.
- The semantics are the same as normal return type deduction and asserting the concept at each return:
  - replace Concept{} with auto;
  - before each return statement, insert static_assert(Concept<R>); where R is the deduced return type.

Notes The static_assert intentionally implies that repeating a function or function template definition with different return type constraints is an ODR violation, even if the repeated signature and body are otherwise identical.
Since C++14 we already have a precedent for the static_assert-like (hard-error) behavior of constrained return types, in contrast to the SFINAE behavior of constrained parameter types, in the semantics for auto*:

```cpp
auto* f(/*...*/) { /*...*/ }    // C++14
```

In C++14, this constrains `f` to return a pointer, and is very similar to constraining a return type with an actual concept for `Pointer`. In C++14, this produces the same static_assert-like behavior, namely a hard error (not SFINAE) if violated.

In the declaration of a leading return type for a function template, permit the constrained type introducer `Concept{ /*T*/ }` as the type. The following rules apply:

- The list of introduced names must have 0 or 1 entries (n=1).
- If the list is empty, the semantics are the same as moving the constraint to the trailing position:
  - change `Concept{T} function_name(/*...*/)` to `auto function_name() -> Concept{T}`.
- If the list is nonempty, then `T` must be used as a parameter type, and the semantics are the same as moving the constraint to the first deduced parameter and then referring to it in the trailing return type:
  - change `Concept{T} function_name(/*...*/, cv-qual T first_t_param, /*...*/)` to `auto function_name(/*...*/, cv-qual Concept{T} first_t_param, /*...*/) -> T`

For example, writing this:

```cpp
void f(Concept{T} a, Concept{U} b) { /* can use the names T and U here */ }
void g(Concept{}) a)               { /* if we don’t need the name, can omit it */ }
void h(Concept{T} a, T b)          { /* same type */ }
```

is equivalent to writing this:

```cpp
template<Concept{T}, Concept{U}> void f(T a, U b) { /* can use the names T and U here */ }
template<Concept{_T}> void g(_T a)               { /* name _T not available to the program */ }
template<Concept{T}> void h(T a, T b)          { /* same type */ }
```

Similarly for generic lambdas, writing this:

```cpp
auto f = []<Concept{T} a, Concept{U}> (T a, U b) { /* can use names T and U here */ };  
auto g = []<Concept{_T}> (_T a)               { /* name _T not available to the program*/ };  
auto h = []<Concept{T}> (T a, T b)          { /* same type */ };  
```

is equivalent to writing this:

```cpp
auto f = []<Concept{T}, Concept{U}> (T a, U b) { /* can use names T and U here */ };  
auto g = []<Concept{_T}> (_T a)               { /* name _T not available to the program*/ };  
auto h = []<Concept{T}> (T a, T b)          { /* same type */ };  
```

### 3.2.2 Discussion

Note that the syntax is now clear about what is a template parameter type in a signature. For example, consider:

```cpp
void constrained_forwarder(X{T}&&);
```

In this proposal, we do not need to go look up `X` to see whether it is a type or a concept to know that this `&&` is a forwarding reference, not an rvalue reference.
Notes

Another way of thinking about it is: If you see an introducer \{T\}, you know T is introducing a name for a type that is being deduced and then constrained.

In the future if we allow omitting empty \{}, then we would have the syntactic ambiguity of the Concepts TS terse syntax regarding whether the parameter is a forwarding reference or an rvalue reference. Some people are concerned about that, some are not. The point of this proposal is that we no longer need to worry about that now, and can defer the decision regarding whether to allow omitting empty \{} until a future time when we have more experience and familiarity with the syntax and may feel comfortable doing so.

Multiple redefinitions flirt of the same function or function template with ODR in the usual ways, with the addition that the static_assert creates an ODR violation if the same function or function template is defined twice with different constraints even if it is otherwise identical, because this:

```cpp
Concept1{} foo(T x) {
    return x;
}
Concept2{} foo(T x) {
    return x; // ODR violation
}
```

is equivalent to this:

```cpp
auto foo(T x) {
    static_assert(Concept1</*type of x*/>);
    return x;
}
auto foo(T x) {
    static_assert(Concept2</*type of x*/>); // ODR violation
    return x;
}
```

This is an ODR violation even if all deduced return types always satisfied both constraints, which they might not.

### 3.3 Part 3: Constrained type names in function bodies

#### 3.3.1 Proposal

For local variable declarations, the semantics are identical to today's auto except that additionally a deduced type can be constrained and (optionally) be named.

For a local variable declaration, permit the constrained type introducer Concept\{/*T*/\} as the type in positions where auto would be allowed today. The following rules apply:

- The list of introduced names must have 0 or 1 entries (n<1).
- If the list is empty, the compiler invents an unspecified unique name for T.
- The semantics are the same as:
  - replace Concept\{T\} with auto;
  - add using T = the deduced type of the variable minus top-level cv- and ref-qualifiers; and
  - add the statement static_assert<Concept\{T\}>; following the variable declaration.
Note  C++11 allows redeclaring the same alias in the same scope:

```cpp
{
    using N = int;
    using N = signed int;  // ok
}
```

So for consistency and simplicity, the design above naturally and intentionally permits this:

```cpp
Number{N} var1 = /*...*/;
// ...
Number{N} var2 = /*...*/;
Iterator{It} var1 = /*...*/;
// ...
RandomAccessIterator{It} var2 = /*...*/;
```

in the same cases using allows, which is when the second introducer re-deduces the same type.

For example, writing this:

```cpp
Concept{T} n = init;  // initializer is required iff T is being newly introduced
```

is equivalent to writing this when the name T has not already been introduced to be visible at this point:

```cpp
auto n = init;
using T = decltype(n);
static_assert(Concept<T>);
```

Note  Each use is treated individually. A use either may be the first to introduce the name T, or may use an already-introduced name T and possibly add new constraints at that point.

If the name is omitted, an invented name is generated, and so writing this:

```cpp
Concept{} n = init;  // initializer is required
```

is equivalent to writing this:

```cpp
auto n = init;
using _T = decltype(n);
static_assert(Concept<_T>);
```

For example, with local variables, introducing a new name to get a new type or else reusing a type by its name:

```cpp
Number{N} n = 42.2;  // N is double, n is a double
Number{M} m = 18;   // M is int, m is an int
Number{} n = 42.2;  // n is a double
Number{} m = 18;   // m is an int
Number{N} n = 42.2;  // N is double, n is a double
N m = 18;  // N is double, m is a double
```

Future simplification  →  Concepts TS terse syntax via a default

As before, in the future we could add: “if there are no introduced names we can also omit {}”.

It is deliberate that if we added that, we would have the current Concepts TS terse syntax for constrained variables. As before, my aim is to construct a syntax that works well now and also gives us a
path to adopting the Concepts TS terse syntax in the future, if the committee gains comfort with viewing empty {} as unnecessary once we gain experience with the feature.

Note Even though C++ already supports new auto(init) and new auto{init}, it does not currently support make_unique<auto>(init) which should be preferred. Therefore this paper is not proposing allowing a constrained new-expression such as new Concept{/*T*/}(init) or new Concept{/*T*/}{init} until we have a way to both deduce and constrain general template arguments including the argument of make_unique. We don’t want to create new reasons to use raw new by giving it capabilities that aren’t supported by the safer smart pointers we want to recommend.

3.3.2 Discussion
Constraints that appear in the bodies of if constexpr blocks that evaluate false are not enforced. In this example, at most one of the nested constraints is enforced:

```cpp
template<class C>
void f(const C& c) {
    Iterator<It> a = c.begin(); /* use a */
    if constexpr (x) {
        RandomAccessIterator<It> b = c.begin(); /* ... */
    } else {
        ForwardIterator<It> b = c.begin();
        /* ... */
    }
}
```

is equivalent to writing this:

```cpp
template<class C>
void f(const C& c) {
    auto n = init;
    using _T = decltype(n);
    static_assert(Concept<_T>);
    auto a = c.begin(); using It = decltype(a); static_assert(Iterator<It>);
    /* use a */
    if constexpr (x) {
        auto b = c.begin(); using It = decltype(b); static_assert(RandomAccessIterator<It>);
        /* ... */
    } else {
        auto b = c.begin(); using It = decltype(b); static_assert(ForwardIterator<It>);
        /* ... */
    }
}
```

Also, this:
Integral{T} a = int16_t(0);
int main() {
    Integral{T} b = int32_t(0);
    if (true) {
        Integral{T} c = int64_t(0);
    }
}

is equivalent to this (i.e., the local T is unaffected by the global ::T which it shadows as if the body of main contained a plain using T = /*...*/):

auto a = int16_t(0); using T = decltype(a); static_assert(Integral<T>);
int main() {
    auto b = int32_t(0); using T = decltype(b); static_assert(Integral<T>); // shadows ::T
    if (true) {
        auto c = int64_t(0); using T = decltype(c); static_assert(Integral<T>);
    } // shadows ::T and (main)::T
}
4 P0694R0 examples side by side

To illustrate, consider the P0694R0 examples in order, showing the examples side by side with the preferred expression of the example in P0694R0 on the left and the proposed version on the right. In some cases the latter uses the terse syntax, and in others the traditional (extended) template syntax.

**Future simplification → Concepts TS terse, introducer, and variable syntaxes via defaults**

Note that in this paper’s proposal, if in the future we additionally allow dropping empty {} and template<> as noted earlier, all Concepts TS examples would be supported identically as in the TS except only for the same-type default on parameters.

### 4.1 sort(range)
From P0694R0 section 1:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>void sort(Sortable&amp;);</td>
<td>void sort(Sortable{}&amp;);</td>
</tr>
</tbody>
</table>

### 4.2 sort(iter, iter)
From P0694R0 section 3.1:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>void sort(Random_access_iterator, Random_access_iterator);</td>
<td>void sort(Random_access_iterator{I}, I);</td>
</tr>
</tbody>
</table>

### 4.3 Variables in a block
From P0694R0 section 3.1, where the Concepts TS has independent-type but following P0694R0 which recommends same-type:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>void use2(auto x, auto y) {</td>
<td>void use2(auto x, auto y) {</td>
</tr>
<tr>
<td>Number xx = f(x);</td>
<td>Number{N} xx = f(x);</td>
</tr>
<tr>
<td>Number yy = g(y);</td>
<td>N yy = g(y);</td>
</tr>
<tr>
<td>}</td>
<td>}</td>
</tr>
</tbody>
</table>

### 4.4 Reusing the concept name in the body
From P0694R0 section 3.1, where the Concepts TS does not support the use of Forward_iterator in the body, and P0694R0 recommends same-type:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>void sort(Forward_iterator b, Forward_iterator e,</td>
<td>void sort(Forward_iterator{I} b, I e,</td>
</tr>
<tr>
<td>Relation&lt;Value_type&lt;Forward_iterator&gt;&gt; r) {</td>
<td>Relation{&lt;&gt;}&lt;Value_type&lt;I&gt;&gt; r) {</td>
</tr>
<tr>
<td>vector&lt;Value_type&lt;Forward_iterator&gt;&gt; v {b,e};</td>
<td>vector&lt;Value_type&lt;I&gt;&gt; v {b,e};</td>
</tr>
<tr>
<td>sort(v);</td>
<td>sort(v);</td>
</tr>
</tbody>
</table>
4.5 Return types
From P0694R0 section 3.3:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output_iterator copy(Input_iterator, Input_iterator, Output_iterator);</td>
<td>Output_iterator copy(Input_iterator, Input_iterator, Output_iterator);</td>
</tr>
</tbody>
</table>

Alternatively, using the trailing return syntax:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto copy(Input_iterator, Input_iterator, Output_iterator) -&gt; Output_iterator;</td>
<td>auto copy(Input_iterator, Input_iterator, Output_iterator) -&gt; Output_iterator;</td>
</tr>
</tbody>
</table>

4.6 Same-type resolution cases
From P0694R0 section 3.3:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value next(Value); Value compute1(Value, Value); // homogeneous op Value compute2(Value, Other_value); // heterogeneous op vector&lt;Value&gt; compute3(vector&lt;Value&gt;);</td>
<td>Value next(Value); Value compute1(Value, Value); // homogeneous op Value compute2(Value, Other_value); // heterogeneous op vector&lt;Value&gt; compute3(vector&lt;Value&gt;);</td>
</tr>
</tbody>
</table>

Alternatively, using the trailing return syntax:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto next(Value) -&gt; Value; auto compute1(Value, Value) -&gt; Value; auto compute2(Value, Other_value) -&gt; Value; auto compute3(vector&lt;Value&gt;) -&gt; vector&lt;Value&gt;;</td>
<td>auto next(Value) -&gt; Value; auto compute1(Value, Value) -&gt; Value; auto compute2(Value, Other_value) -&gt; Value; auto compute3(vector&lt;Value&gt;) -&gt; vector&lt;Value&gt;;</td>
</tr>
</tbody>
</table>

4.7 find()
From P0694R0 section 3.4:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input_iterator find(Input_iterator, Input_iterator, Equality_comparable&lt;Value_type&lt;Input_iterator&gt;&gt;&gt;);</td>
<td>Input_iterator find(Input_iterator, Input_iterator, Equality_comparable&lt;Value_type&lt;Input_iterator&gt;&gt;&gt;);</td>
</tr>
</tbody>
</table>

Alternatively, using the trailing return syntax:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto find(Input_iterator, Input_iterator, Equality_comparable&lt;Value_type&lt;Input_iterator&gt;&gt;&gt;)) -&gt; Input_iterator;</td>
<td>auto find(Input_iterator, Input_iterator, Equality_comparable&lt;Value_type&lt;Input_iterator&gt;&gt;&gt;)) -&gt; Input_iterator;</td>
</tr>
</tbody>
</table>
4.8 merge()

Only single-type concepts fit in the terse syntax, because the terse syntax is inherently for constraining a single type. This proposal allows multi-type concepts to work fine in \texttt{template<>} parameter lists.

From P0694R0 section 3.4:

<table>
<thead>
<tr>
<th>P0694R0 style (using introducer syntax)</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{Mergeable{In1,In2,Out}}</td>
<td>\texttt{template&lt;\texttt{Mergeable{In1,In2,Out}}&gt;}</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>\texttt{Out merge(In1, In1, In2, In2, Out);}</td>
<td>\texttt{Out merge(In1, In1, In2, In2, Out);}</td>
</tr>
</tbody>
</table>

4.9 Different-type resolution: operator+() for Numbers

Here P0694R0 cannot use its terse syntax because it would use same-type resolution, and must resort to either a verbose traditional \texttt{template<>} syntax (which is not semantics-preserving because it changes deducibility) or invent duplicate concept names (which feels like a hack).

From P0694R0 section 3.6:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{template&lt;Number N1, Number N2 = N1, Number N3 = N1&gt; N3 operator+(N1,N2);}</td>
<td>\texttt{Number{} \texttt{operator+(Number{}, Number{});}</td>
</tr>
</tbody>
</table>

Note that in this case the left and right versions are not quite equivalent. The left-hand side does not deduce for \texttt{operator+()}\texttt{(\{}x\texttt{)} but does deduce for \texttt{operator+()y\{}\texttt{)}, so it is not actually a general method to get independent-type resolution if the default is same-type.

Alternatively, P0694R0 suggests giving the concept a different name, which proliferates one-off names as a workaround:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{template&lt;typename T&gt; concept Number2 = requires Number&lt;T&gt;;} \texttt{template&lt;typename T&gt; concept Number3 = requires Number&lt;T&gt;;} \texttt{Number3 operator+(Number,Number{}\texttt{2};}}</td>
<td>\texttt{Number{} \texttt{operator+(Number{}, Number{});}</td>
</tr>
</tbody>
</table>

4.10 Multiple constraints: merge\_then\_sort (from §3.1.2)

Here P0694R0 cannot use its terse syntax, and must resort to either a verbose traditional \texttt{template<>} syntax or invent duplicate concept names.

From this paper, §3.1.2:

<table>
<thead>
<tr>
<th>P0694R0 style, using requires</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{template &lt;typename In1, typename In2, typename Out&gt; \texttt{requires Mergeable{In1, In2, Out} &amp;&amp; SortableIterator{Out}&gt; \texttt{void merge_then_sort \texttt{(In1 first1, In1 last1, In2 first2, In2 last2, Out out);} }}</td>
<td>\texttt{template &lt;\texttt{Mergeable{In1, In2, Out}, SortableIterator{Out}&gt; void merge_then_sort \texttt{(In1 first1, In1 last1, In2 first2, In2 last2, Out out);} }}</td>
</tr>
</tbody>
</table>
Alternatively, P0694R0 suggests giving the concept a different name, which proliferates one-off names as a workaround; this pollutes namespaces and is otherwise undesirable:

<table>
<thead>
<tr>
<th>P0694R0 style, using introducer syntax</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>template &lt;typename In1, typename In2, typename Out&gt; concept MergeableAndOutIsSortable = Mergeable&lt;In1, In2, Out&gt; &amp; SortableIterator&lt;Out&gt;;</td>
<td>template &lt;Mergeable{In1, In2, Out}, SortableIterator&lt;Out&gt;&gt; void merge_then_sort (In1 first1, In1 last1, In2 first2, In2 last2, Out out);</td>
</tr>
</tbody>
</table>

### 4.11 Constraints on both type and non-type parameters

In the Concepts, but not the current C++20 draft, a constraint can be applied to a non-type parameter and the meaning depends on the implementation of the concept. Constraining a non-type parameter uses an inconsistent special meaning that is different than constraining a parameter or local variable.

<table>
<thead>
<tr>
<th>Concepts TS</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>template&lt;Number Type, Numeric Value&gt; void f(Number parameter) { Number variable; }</td>
<td>template&lt;Number&lt;Type&gt;, Number{} Value&gt; void f(Number{} parameter) { Number{} variable; }</td>
</tr>
</tbody>
</table>
5 Discussion / Q&A

5.1 Why are unnamed constrained types independent?
This section addresses the question: If we introduce multiple unnamed constrained type names, should their types be the same, or be independent?

5.1.1 Proposal recap
As already noted, in this proposal, when an introduced type name is unnamed, the compiler invents an unspecified unique name. This is consistent with what we do in the rest of the language for unnamed entities (e.g., lambdas). Therefore, writing this:

```cpp
void g(Concept {} a, Concept {} b) { /* if we don’t need the names, can omit them */ }
```

is equivalent to writing this, where _T1 and _T2 are unique names invented by the compiler:

```cpp
void g(Concept {_T1} a, Concept {_T2} b) { /* ... */ }
```

Similarly for generic lambdas, writing this:

```cpp
auto g = [](Concept {} a, Concept {} b) { /* if we don’t need the names, can omit them */ };  
```

is equivalent to writing this:

```cpp
auto g = [](Concept {_T1} a, Concept {_T2} b) { /* ... */ };  
```

Because introducing different (user-written) type names introduces independent types, anonymous types with unique invented names are naturally independent too. I believe this consistency, together with the resulting natural expressiveness in the list of side-by-side examples in §4, is sufficient to justify independent-type semantics for unnamed constrained types. However, additional reasons for independent-type resolution appear below.

5.1.2 General problems with Concepts TS same-type default
The Concepts TS terse syntax requires same-type semantics for some cases, and P0694R0 argues for changing the TS semantics to add more cases. For example:

```cpp
auto f(Concept a, Concept b) -> Concept  
    requires Something<Concept>  
{  
    some_template<Concept> x = /*...*/;  
}  

auto g() {  
    Concept xx = f();  
    Concept yy = g();  
}  
```

This has several serious problems:

- **Consistency**: Whenever we do require same-type, we change semantics. For example, if the parameter types were `auto` or variadic `Concept`... then they would be independent in the Concepts TS; changing
semantic meaning between (Concept,Concept) and (Concept...), or between (Concept,Concept)
and (auto,auto), is inconsistent. Making the list variadic should not change its basic semantics, and
using auto as the least constrained “satisfied by any type” concept should not change the constraint’s
basic semantics.—As another example, if the return type were unconstrained, or did not use the same
constraint as a parameter, the return type would be deduced from the return-expression; but if it hap-
pens to be constrained, and then only with a constraint also used to constrain a parameter, it is no
longer deduced; I feel that too is inconsistent.

- Incomplete / slippery slope: Once we begin to implicitly say “some” uses of the same concept name
must be the same type, but “not others,” we will continue to find reasons to add more cases, as shown
above. The Concepts TS added some same-type cases; P0694’s main text proposes adding new cases,
and its final section suggests exploring still more. Even if we want to pursue same-type, we’re clearly not
done specifying when we want it. This is a moving target where each addition is an ad-hoc patch.

- Teachability / usability: Teaching “sometimes using a concept name twice means same-type, some-
times it doesn’t” is subtle—you “just have to know” and teach the rules by rote.

This paper advocates the core principle that **adding a constraint should not change the meaning of code**—there
should be no change in semantics, only rejecting uses of a type that does not match the constraints. This princi-
ple is at the root of most of the (good) technical concerns raised above and in **P0464R2**: Violating this principle
(by additionally saying that “in some cases” multiple uses of the concept are also the same type) is what leads to
inconsistency with auto, inconsistency with dynamic polymorphism, inconsistency with variadic templates, in-
consistency with return type deduction, and other problems.

5.1.3 Stroustrup’s empirical STL analysis strongly favors independent-type default
For **P0694R0** Stroustrup did extensive and important research work, multiple times, to carefully and thoroughly
compile data and statistics from analyzing the current STL; aside from a similar analysis of the Ranges TS by its
authors, no other paper authors, myself included, have made such a rigorous effort, and that effort and data is
much appreciated—and should be given more weight than design papers with less data and therefore neces-
sarily based more on personal design sensibility and conjecture.

Consider that research data from analyzing the STL. The same-type terse default was motivated by the observa-
tion that the current STL algorithms’ parameters are dominated by [first,last) iterator pairs. However, iterator
pairs are known to be a design flaw—any time we have two variables that share an invariant, we know we
are missing an abstraction (by definition, because an invariant should be encapsulated within a single object).
Thankfully that is being addressed with ranges; and, in part at my request during draft review of D0694R0 (and
with great thanks), in P0694R0 Stroustrup additionally took time to perform a careful analysis of what the STL
data would look like if we made just the single change of removing iterator pairs with a sequence (range) ab-
straction, and no other change. The results reported in P0694R0 are as follows (highlights added):

```
4.2 Using a sequence abstraction
The pair-of-iterators idiom is largely responsible for the high number of repeated concepts. A roughly equivalent library based on a sequence idiom (where a sequence would be an object holding a pair of iterators or equivalent), the numbers would be:

- 173 with no repeated concepts
- 0 where all repeated concepts must be the same
- 78 where all repeated concepts can be different
- 0 where some repeated concepts can differ and some duplicate concepts must be the same
```
The initial conclusion from this data, as reported in P0694R0, was that 69% of the algorithms (category #1) are easily expressed using the terse syntax with same-type semantics. However, those same 69% would equally have been expressed without same-type semantics because by definition they contain no repeated concepts. A better characterization is that “only 69%” can be expressed in the terse syntax using the same-type default.

I believe the stronger conclusions from this data set are that:

- **100%** can be easily expressed using the terse syntax with independent-type semantics;
- **31% (78)** require independent-type semantics to be easily expressed using terse syntax; and
- **None** actually benefit from same-type semantics as the default.

**Note** The “real” sequence-ized STL is the Ranges TS, which as reported in P0694R0 cannot use the terse syntax at all with same-type semantics (and likely not even with independent-type semantics).

Of course, everyone agrees that that there are examples that want to express same-type and examples that want to express independent-type; both need to be expressible (see especially §5.1.4 below). However, in the context of the empirical analysis of the STL, it is worth emphasizing that the above is the result after STL iterator pairs are removed—that is, the only class of example in our empirical data so far presented to argue in favor of the Concepts TS same-type as a default is STL iterator pairs. And even that example is simpler to express in this proposal (even with independent-type default) than in the Concepts TS with its same-type default:

<table>
<thead>
<tr>
<th>Concepts TS (using same-type default)</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>void sort(Iterator first, Iterator last);</td>
<td>void sort(Iterator(I) first, I last);</td>
</tr>
</tbody>
</table>

The TS design (left) has several problems, including that it is:

- more implicit and therefore opaque, because programmers must learn and remember that this is one of the cases where mentioning the same concept name twice imposes a same-type requirement so that `first` and `last` must be the same type;
- inconsistent, because mentioning the same concept name again does not have same-type semantics in other local uses; and
- slightly more verbose, because of the implicit redundancy in meaning.

I feel that this paper’s proposed design (right) is better because it is:

- explicit and clear, because it is obvious that `first` and `last` have the same type for the simple and usual reason that they use the same type name;
- consistent, because in the function template the same name always implies the same type and a different name always implies an independent type; and
- less verbose, because it removes the implicit redundancy in meaning.

### 5.1.4 Complexity of opting out favors independent-type default

Further, even if the examples for same-type and independent-type were otherwise equally desirable (which I believe is not the case), whichever we choose as the default will require us to opt out when we want the other. With same-type as the default, to opt out of N uses of the concept to make them independent-type requires writing N names, whereas with independent-type as the default, to opt out of N uses of the concept to make them the same requires writing one name:
// Using Concepts TS syntax
// given this, with some default meaning (either same-type or independent-type)
Concept f(Concept, Concept, Concept);

// if same-type is the default and we want to opt to independent-type
template<Concept C1, Concept C2, Concept C3, Concept C4> C1 f(C2, C3, C4);

// if independent-type is the default and we want to opt to same-type
Concept{C} f(C, C, C);

Especially given that we have examples that benefit from same-type and examples that benefit from independent-type with no conclusive data that either is dominant in general use, optimizing the design to ‘give the nice syntax to the common case’ should include choosing the default that minimizes the overall opt-out burden.

5.1.5 This proposal: Convenient choice of either same-type or independent-type

In this proposal, returning to the earlier example, if you want same type then give the type a name and use its name:

// Proposed semantics: If you want same type, name the type and use the name
auto f(Concept{T} a, T b) -> T // same types
    requires Something<T> // same type
{
    some_template<T> x = /*...*/; // same type
}

Note This seems about as concise a way to apply a concept constraint as possible over ordinary syntax; it simply names a type in-place and keeps going. Not that terseness is a virtue in itself, but this is by far the tersest way to say this in any of the alternatives proposed in papers so far including the Concepts TS terse syntax (and similar in terseness to [Van17]).

auto g() {
    auto N xx = f();
    N yy = g(); // same type
}

And if you want independent types, either give them different names, or don’t name them if no name is needed:

// Proposed semantics: If you don’t want same type, give them all different names
// or omit the name(s) that don’t need to be referred to again
auto f(Concept{} a, Concept{} b) -> Concept{} // independent types
    requires Something<Concept> // (n/a for independent type)
{
    some_template<Concept> x = /*...*/; // independent type
}

auto g() {
    auto Number{} xx = f();
    Number{} yy = g(); // independent type
}
Notes
It is intentional that these rules work the same as if we had used the least constrained concept, auto.

The delta from today’s terse syntax is just {}, and doubtlessly sooner or later someone will propose dropping the “unnecessary empty {}”—it is unclear whether the committee will ever be comfortable with this, but if it did adopt this we would be back to exactly the Concepts TS terse syntax except with independent-type resolution. Leaving this option available, while not using it immediately, is an explicit design goal—because of this, I believe that this paper’s approach may be our best chance for (a) giving those people who are uncomfortable with zero syntactic differences what they want for now (i.e., a syntactic marker) while (b) keeping that difference very lightweight and (c) also leaving the door wide open to erasing that tersely later if and when we as a group gain familiarity and comfort with unifying generic and ordinary programming.

One design question is whether Concept{T} should allow default type arguments, e.g., Concept{T = vector<int>}. In parameters, the main use case that we have been able to find so far is if we want the caller to be able to pass {} as an argument, or if the parameter has a defaulted argument of {} or T{}—but the latter is predominant in the Ranges TS for projection and comparison operators. If we supported this, then the Ranges TS could change this

```cpp
template <Sortable{I, Comp = less<>, Proj = identity}>
I sort(I first, S last, Comp comp = Comp{}, Proj proj = Proj{});
```

to this:

```cpp
template <Sortable{I, Comp = less<>, Proj = identity}>
I sort(RandomAccessIterator{I} first, Sentinel<I>{} last, Comp comp = Comp{}, Proj proj = Proj{});
```

Summarizing side by side with the P0694R0 preference:

<table>
<thead>
<tr>
<th>P0694R0 style</th>
<th>This paper (proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto f(Concept a, Concept b) -&gt; Concept // implicitly same types requires Something&lt;Concept&gt; // P0694: same type</td>
<td></td>
</tr>
<tr>
<td>{}</td>
<td></td>
</tr>
<tr>
<td>Something&lt;Concept&gt; x = /<em>...</em>/; // P0694: same type</td>
<td></td>
</tr>
<tr>
<td>auto g()</td>
<td></td>
</tr>
<tr>
<td>Number xx = f();</td>
<td></td>
</tr>
<tr>
<td>Number yy = g(); // P0694: same type</td>
<td></td>
</tr>
</tbody>
</table>

5.2 Why no multi-type constraints in single-type contexts?

5.2.1 Parameters, return values, and local variables
This proposal deliberately does not propose allowing multi-type constraints on parameters, return values, or local variables, where the language by construction is talking about only a single type.

There is a temptation to try to extend declarations of multiple actual parameters/variables to allow multi-type constraints, but I could not find a way to do it naturally that did not make readability suffer. For example, I considered allowing a generalized multi-type parameter syntax that allows something like this:
void f(Concept{X x, Y y}); // not proposed

as a synonym for this:

template<Concept{X, Y}> void f(x, y); // hypothetical meaning

However, for both parameters and true local variables we would then have to support at least default arguments/initizers and possibly cv-qualifiers too, such as:

void f(Concept{X x, Y y = y_init})

and the variable case where the initializer is mandatory since otherwise we have no type to deduce:

Concept{X x = x_init, Y y = y_init};

I worry that this last is quickly losing readability, and no longer looks like two local variable declarations.

Furthermore, to make this broadly useful, the order and number of type constraints in a concept and parameters or variables constrained by that concept would need to commonly agree, but examples where they do are scarce and un compelling; instead, those orders and numbers are generally unrelated.

I do not view the restriction of using only single-type constraints in single-type contexts as a loss of generality in this proposal. In this proposal, multi-type constraints are universally allowed in all places where multiple types can exist or be introduced; and if the language in the future allows more places where multiple types can be named, such as multi-type variable introductions, then multi-type constraints can naturally be applied there too.

5.2.2 Structured bindings

This proposal also deliberately does not propose allowing multi-type constraints on structured bindings.

There is a temptation to try to allow N-ary multi-type constraints on N-ary structured bindings, such as:

Concept{X, Y, Z}[x, y, z] = init();

However, this apparent symmetry seems illusory, because again in general the order and number of type constraints is unrelated to the order and number of components of an aggregate, and the cases that “work” are usually forced instead of expressing something that programmers actually want to express.

However, as a future pure extension of this proposal, we could consider allowing single-type constraints on individual elements of a destructuring, for example:

auto [Number{X} x, y, Iterator{} z] = init();

which does not cause the problems that allowing a concrete type in that position would cause (i.e., the type is still deduced, there are no conversions, there is no pressure to additionally allow other things like cv-qualifiers).

5.3 Can we have a template prefix on function templates?

The proposed syntax already makes function templates visually distinct via {}, which addresses one of the objections to the original Concepts TS syntax.

A small number of reviewers of this paper have suggested additionally requiring the unadorned keyword template, without a parameter list, in front of function template declarations to make them more visually distinct. I worry about this for several reasons: One is that it treads very near the existing syntax we have for explicit instantiations. Another is that I don’t think it has the desired benefits. The basic argument I’ve heard about why it may be desirable for the word “template” to be required throughout the language when writing a template is
twofold: (a) for consistency, and (b) to make it clearer still that function template rules apply in the body rather than normal function body rules. I do not find this argument for additionally requiring the word “template” convincing: for (a), the consistency argument fails because “template” is already not required for generic lambdas, which are (or generate) function templates; and for (b), not only do the {} in this proposal make templates visible, but additionally field experience over the past half-decade with C++14 generic lambdas (which also are visually distinct, with the keyword auto) has not brought any feedback that I am aware of to the effect that the absence of the word “template” somehow makes generic lambdas more difficult or brittle in practice. With generic lambdas, the presence of auto is sufficient unambiguous indicator that we are writing a template, and in this proposal the presence of {} is also a sufficient unambiguous indicator.

However, if and only if this proposal as written does not manage to get consensus, and just adding template would by itself make the difference in this proposal getting consensus, then we could tack on a “required template” wart that, like empty {}, we can consider removing later once we gain more familiarity with the feature. The result would be that all examples in this proposal are identical, with the exception of adding the boilerplate word, such as:

```cpp
template void sort(Sortable{}&);
template void sort(Random_access_iterator{}, I);
```

If we did add this keyword requirement, we could already anticipate the arguments in the future paper to remove the redundant template: that it adds no information (the code is unambiguous without it), that it is tedious boilerplate (a case of the compiler saying “I know exactly what you mean but I’m going to force you to write an extra word,” one of the kinds of error messages most resented by C++ developers), and that removing it increases consistency in the language (with generic lambdas).

5.4 What about “value concepts” and “adjective syntax”?  
Recall from §2.2.4 that in this proposal, unlike the Concepts TS and the C++20 working draft, we naturally support using the same concept to constrain both a type parameter and non-type parameter (the latter with the same syntax and semantics as value parameters and variables).

Some experts, including in Issaquah 2016 (Richard Smith) and papers P0791R0 (Riedle) and P0807R0 (Köppe), have suggested a generalization whereby we might use concepts to express both type constraints and value constraints, and because types and values can lexically appear in similar positions to have a syntax that requires typename for constrained types and auto for constrained variables, with concepts allowed as adjective prefixes to both. That is, to have Concept typename T constrain a type T, and Concept auto v constrain a variable v and be termed a “value concept.” It is an interesting idea, though as the papers note it has not been deeply explored.

The immediate problem with such an approach is that it requires more tedious boilerplate (typename and auto) that is redundant in all current examples compared to this paper, in the name of a generalization that is unproven so that we can’t evaluate whether it would really deliver more consistency throughout the language or the ability to express things that are harder or impossible to express today.

The more fundamental problem I see with this approach, and why I doubt the generalization can ever be a good fit for concepts, is that an object’s type and its values are fundamentally distinct things:

- An object’s type is invariant for the lifetime of the object. Therefore a type constraint naturally should appear on the type (i.e., on the object’s declaration), and is a good match for a constraint that is logically
and lexically/visually applied as a type modifier, such as to state that variable \(n\) is always a **Number** for its entire existence.

- An object’s value is a dynamic property that can change over the lifetime of the object, even for purely compile-time (and so “static” in a different sense) objects such as local variables in `constexpr` functions used to initialize `constexpr` values. Therefore a value constrain naturally should appear on the value (i.e., on expressions and statements that manipulate the value, including but not limited to the initial value at time of initialization which could happen to usually appear near the object declaration), and is not a good match for a constraint that is logically or lexically/visually attached to the object’s type (e.g., a numeric variable \(n\) generally is not always an **Even** number at all points of its lifetime, even if its initial values is even). Fortunately, we are already progressing the proposal that does provide a match, which is constraint via general contracts.

In short, concepts are to constrain types and naturally should annotate the type (i.e., object declarations), whereas contracts are to constrain values and naturally should annotate the values (i.e., expressions and statements). Because those two things have fundamentally different properties, I suspect any attempt to conflate them will make code less clear, not more clear.

For example, consider this example which appears in both P0791 and P0807 (updated to current WP syntax):

```cpp
template<int N> concept Even = N%2 == 0;
```

and the proposed use as shown in those papers, contrasted what we would write with contracts (P0542):

```cpp
// P0791/P0807 proposal: ‘Even’ is a type modifier (invariant property of i)
void f(Even int i);
void g(Even auto i);

// P0791/P0807 proposal: ‘Even’ is a type modifier (invariant property of i)
void f(Even int i) {
    i += 2;       // P0791 intends this be allowed
    i += 3;       // P0791 intends this be a constraint violation
}

// P0791/P0807 proposal: ‘Even’ is a type modifier (invariant property of i)
void f(Even int i) {
    i += 2;       // P0791 intends this be a constraint violation
    i += 3;       // P0791 intends this be a constraint violation
}
```

So far both may look plausible, though the contract syntax has the important property that it is logically and visually distinct from \(i\)’s type. This is makes it already more clear, more correct, and more general, because we see that **is_even** is a property of the value at a point in time only, not an invariant statement about \(i\)’s nature at all times.

But next, look into the function body: What if we wrote `++i` in \(f\)’s body—should that be allowed? The answer in P0791 is no, through dynamic checking (the answer in P0807 appears to be that this was not considered because the constraint was intended to be limited to checking an immutable value such as a template non-type parameter):

```cpp
// P0791 proposal: ‘Even’ is a type modifier (invariant property of i)
void f(Even int i) {
    i += 2;       // P0791 intends this be allowed
    i += 3;       // P0791 intends this be a constraint violation
}

// P0542 contracts: ‘is_even’ is a check of i’s value on entry
void f(int i) [[expects: is_even(i)]] {
    i += 2;       // ok
    i += 3;       // ok
```
Finally, let’s rename \( f \) to \( \text{make_odd} \) (a function that takes an even \( \text{int} \) and returns an odd one):

\[
\text{Odd int make_odd(}\text{Even int i}) \{
    \text{++i; // } \text{P0791 intends that this should be illegal, but that}
    \text{return i; // “return i+1;” be allowed, which feels like a programming}
\}
\]

// call site: \( \text{Odd int i = } f(2); \text{ ++i; // disallowed, same issue again here} \)

// P0542 contracts: ‘is_even’ and ‘is_odd’ look like a check of \( i \)’s value on entry
\[
\text{int make_odd(int i) }[[\text{expects: is_even(i)}]] [[\text{ensures: is_odd(i)}]] \{
    \text{++i; // ok (abd “return i+1;” would be ok too)}
    \text{[assert: is_odd(i)]}; // alternative position for check}

\]

With the second contract form, it’s natural and logically clear that \( \text{is_even} \) is a property of the initial argument value on entry, and \( \text{is_odd} \) is a property of the returned value. I do not see a way that trying to wedge this information into a type modifier makes code clearer; though of course we could contort the language to force it to ‘work,’ I think any attempt to paper over this impedance mismatch will necessarily interfere with readability.

So I strongly support value constraints, but I think the generalization is to contracts (which already support this and so the generalization is already complete and done) rather than concepts (by inventing a novel attempted generalization that makes it appear to be an invariant property of the type when it is not).

**Note** I have corresponded at length with the author of P0791R0 (Riedle) and he understands this. It turns out the motivation for P0791R0 is not about concepts per se, but rather that the author objects to the contracts proposal P0542 on the grounds that the contracts are too low-level (granular code in attributes) and can be disabled (by changing contract checking levels), whereas he feels it is essential that contracts be able to be pushed up to the call site.

In competition with P0542 contracts, the intention of P0791R0 actually is to try to force contracts instead into the type system as type qualifiers so that they are part of the signature of a function, that callers cannot turn them off, and that they be pushed to the call site. The intention is also to enable powerful Haskell-like advanced overloading such as this:

\[
\text{// Example provided by J. Riedle: The goal is that overload resolution}
\text{// would see that an input value is Empty and if so call the no-op}
\text{auto quick_sort( Empty Range auto& rng )} \{ \text{return std::move(rng); } \}
\text{auto quick_sort( NonEmpty Range auto rng )} \{ \text{/**/; } \}
\]

and enable optimization based on run-time values such as this:

\[
\text{// Example provided by J. Riedle: The goal is that if the input matrix}
\text{// is found to be already Symmetric, this function can become a no-op}
\text{concept_cast<Symmetric matrix>( my_matrix )};
\]

These ideas may be interesting, though I have concerns about the viability of this approach (for example, it would require introducing a dynamic component to overload resolution which is certainly possible but that we do not have today and would require careful thought), but I don’t think they
should influence our concept constraint design because I don’t believe anything in this proposal rules out pursuing such a direction in the future, while on the other hand I don’t think this direction is sufficiently developed to justify pursuing a more verbose concept constraint design now in anticipation of this approach maturing soon.

One reviewer asked: “I’m having trouble squaring these thoughts with section 2.2.1 of your proposal in which you retain the ability to constrain a non-type template parameter via use of a concept as a type specifier.”

The answer is that that’s totally different—in 2.2.1 it’s just a normal type concept and type constraint. Consider:

```cpp
// unconstrained
template<auto Size>  
void f() {    
   auto num = /*...*/;
} 

// constrained (with: template<typename T> concept Number)  
template<Number{} Size>  // type concept  
void f() {   
   Number{} num = /*...*/;   // type concept
}
```

In this proposal, those both do the same thing, they’re the identical type concept being applied with identical semantics to constrain the deduced types of Size and num.

In contrast, a value concept is not the same thing at all and leads instead to this:

```cpp
// constrained (with: template<typename T> concept Number, template<int T> concept Even) 
template<Even Size>  // value concept 
void f() {    
   Number num = /*...*/;   // type concept
}
```

which is very different because the first constraint tries to constrain the value, not the type, of Size (and we want, and should have, a way to constrain the type of Size). I think that the discussion in this section applies equally to value concepts as being not a useful direction, because concepts are not the right tool to constrain values (and even if they did exist they especially should not visually appear to be applied to type as in the TS) for the same reasons as (and that appear to have led directly to) the adjective concepts proposals discussed in this section. Separately from this proposal, we should consider removing value concepts as an undesirable direction.

### 5.5 Concept{T1, T2} vs. Concept[T1, T2]

This paper shows `{}` syntax because it’s the least delta from the Concepts TS syntax.

It has been suggested that `{}` is getting overused, and that we could instead follow structured bindings’ precedent for introducing names using `[ ]`. Example: `Mergeable[X, Y, Z]` for symmetry with `auto [x, y, z] = f();`.

**Notes** Structured bindings was also initially proposed with `{}` syntax, but EWG preferred `[ ]` syntax.

I’m not aware of any current ambiguity that would be caused by using `[ ]`. For example, array parameters would be `Concept[T] a[]` where the `[ ]` do not collide, although they have different meanings. And a function returning an array of constrained type would be `Concept[X][] f();` which appears unambiguous.
However, two different reviewers opined that “Concept[] looks like how array declarations should look, but don’t (e.g., int[10] x);” so that [] is more likely to interfere with making the language more regular in the future, and may already create visual confusion for some readers.—Also, if in the future we want to explore constraining structured bindings in the position where auto goes today to constrain the whole unnamed composite entity (despite the rationale given in §5.2.2), then using [] (e.g., Concept[X] x; for a variable vs. Concept[X,Y,Z][x,y,z] for a destructuring) means parsing starts to need more lookahead (or lookup to disambiguate, to determine Concept is a concept) whereas {} would not.

If we followed the same here, the examples in §4 would look as follows:

```cpp
void sort(Sortable[] &);
void sort(Random_access_iterator[I], I);
void use2(auto x, auto y) {
    Number[N] xx = f(x);
    N yy = g(y);
}
void sort(Forward_iterator[I] b, I e, Relation[]<Value_type<I>> r) {
    vector<Value_type<I>> v {b,e};
    sort(v);
    copy(v.begin(),v.end(),b);
}
Output_iterator[O] copy(Input_iterator[I], I, O);
Value[V] next(V);
Value[V] compute1(V,V);     // homogeneous op
Value[V] compute2(V,Other_value[]); // heterogeneous op
vector<Value[V]> compute3(vector<V>);
    auto next(Value[V]) -> V;
    auto compute1(Value[V],V) -> V;
    auto compute2(Value[V],Other_value[]) -> V;
    auto compute3(vector<Value[V]>) -> vector<V>;
Input_iterator[I] find(I, I, Equality_comparable<Value_type<I>>[]);
    auto find(Input_iterator[I], I, Equality_comparable<Value_type<I>>[]) -> I;
    template <Mergeable[In1,In2,Out]>
    Out merge(In1, In1, In2, Out);
Number[] operator+(Number[], Number[]);
    template <Mergeable[In1, In2, Out], SortableIterator[Out]>
    void merge_then_sort(In1 first1, In1 last1, In2 first2, In2 last2, Out out);
    template<Number[Type], Number[] Value>
    void f(Number[] parameter) {
        Number[] variable;
    }
```
6 Bibliography

The following are key “landmark” historical concepts papers, and most current concepts papers, in chronological order. Note: P0557R0 and P0694R0 are considered prerequisites to reading this paper.


[P0464R2] T. Van Eerd, B. Ballo. “Revisiting the meaning of ‘foo(ConceptName,ConceptName)”’ (WG21 paper, 2017-03-12).


[P0694R0] B. Stroustrup. “Function declarations using concepts” (WG21 paper, 2017-06-18). A comprehensive and thorough treatment (and not the first, but the most recent)—prerequisite reading assumed to be understood before attempting this paper.


