Reducing <ratio>

Abstract

Using header <ratio> today, the ratio 1 : 2 could be represented by any of a potentially large number of distinct types: ratio<1,2>, ratio<2,4>, ratio<5,10>, etc. Algorithms must therefore be constantly aware that any particular ratio type may be in unreduced form.

As promised in [P0548R0], this paper proposes to adjust the specification of those <ratio> types corresponding to unreduced ratios so that they become aliases for the unique type corresponding to that ratio in reduced form. (E.g., ratio<2,4> and ratio<5,10> would each alias ratio<1,2>.) This will reduce cognitive burden on programmers and can also lead to somewhat simpler code.

The ratio of something to nothing is infinite. So just do something.
— Peter Diamandis

The only true measure of success is the ratio between what we might have done and what we might have been on the one hand, and the thing we have made and the things we have made of ourselves on the other.
— Herbert George “H.G.” Wells

There’s a fine line between a numerator and a denominator. (Only a fraction of people will find this funny.)
— Unknown

1 Background and proposal

Elementary school children are routinely taught how to reduce a fraction to its lowest terms: divide both its numerator and its denominator by their greatest common divisor. Children are also taught that ratios are commonly expressed as fractions, and that ratios can therefore be similarly reduced.

When the <ratio> header was first designed (circa 1999), we decided (a) to allow any representable ratio (whether reduced or unreduced) to be expressed, and (b) to provide a type alias member and corresponding static data members denoting the ratio’s reduced equivalent. This design resulted in the specification that is today found in subclause [ratio.ratio] of the post-Kona Working Draft [N4659]:
namespace std {
    template <intmax_t N, intmax_t D = 1>
    class ratio {
    public:
        static constexpr intmax_t num;
        static constexpr intmax_t den;
        using type = ratio<num, den>;
    };
}

1 If the template argument \( D \) is zero or the absolute values of either of the template arguments \( N \) and \( D \) is not representable by type \( \text{intmax}_t \), the program is ill-formed. [Note: These rules ensure that infinite ratios are avoided and that for any negative input, there exists a representable value of its absolute value which is positive. In a two’s complement representation, this excludes the most negative value. — end note]

2 The static data members \( \text{num} \) and \( \text{den} \) shall have the following values, where \( \text{gcd} \) represents the greatest common divisor of the absolute values of \( N \) and \( D \):

\[
(2.1) \quad \text{num} \text{ shall have the value } \text{sign}(N) \times \text{sign}(D) \times \text{abs}(N) / \text{gcd}.
\]

\[
(2.2) \quad \text{den} \text{ shall have the value } \text{abs}(D) / \text{gcd}.
\]

In that C++98 era, this seemed the best we could do. Alas, these decisions regrettably allowed a potentially large number of types to denote the same notional value: for example, the ratio \( 1 : 2 \) could be represented by the types \( \text{ratio}<2,4> \), \( \text{ratio}<5,10> \), \( \text{ratio}<1,2> \), or any of many, many others. Thus, \( \langle \text{ratio} \rangle \) users must take into account that any particular \( \text{ratio} \) type may be in unreduced form. Unfortunately, this consequence is sometimes overlooked. Even when taken into account, it has led to convoluted and error-prone code.

Via aliases, however, modern C++ allows us to ensure that such currently-distinct types \( \langle \text{ratio}<2,4> \rangle, \langle \text{ratio}<5,10> \rangle, \langle \text{ratio}<1,2> \rangle \), etc.) all denote a single type, namely the equivalent reduced type \( \langle \text{ratio}<1,2> \rangle \). As illustrated in the next section (and as had been promised in [P0548R0]), this paper proposes to adjust \( \langle \text{ratio} \rangle \)'s specification so as to do just that, and thus obtain only reduced \( \langle \text{ratio} \rangle \) types.

This change will simplify reasoning about types (such as \( \langle \text{std::duration} \rangle \)) that rely on \( \langle \text{ratio} \rangle \). It may also somewhat simplify their implementation, as they no longer need concern themselves with unreduced \( \langle \text{ratio} \rangle \) types. Moreover, it will simplify specification of ratio arithmetic, as shown in §3, by removing extra wording to require reduced results.

## 2 Expository implementation

In brief, our strategy here\(^1\) is (a) to rename today's \( \text{ratio} \) as, say, \( \_\text{ratio} \), and then (b) to redefine \( \langle \text{ratio}<N,D> \rangle \) as an alias template for the equivalent \( \_\text{ratio}<N,D>:\text{type} \). (We do not show the helpers \( \text{num\_min\_v, static\_abs\_v, static\_gcd\_v, and static\_sign\_v} \), as we believe their intent is as obvious as their implementation is straightforward.)

\[
\begin{align*}
\text{template} & < \text{intmax}_t N, \text{intmax}_t D > \\
\text{class } & \_\text{ratio} \ {\text{ // same members as the former ratio template}} \\
& \text{static\_assert}( \ N \neq \text{num\_min\_v<\text{intmax}_t> } ); \\
& \text{static\_assert}( \ D \neq 0 \ \text{and} \ D \neq \text{num\_min\_v<\text{intmax}_t> } );
\end{align*}
\]

\( \text{See } \S 4 \text{ for an alternate approach.} \)
We validated the above updated implementation against the same public and private tests used to validate implementations of the present specification. Every test passed unmodified.\footnote{There were no tests expecting \texttt{is\_same\langle\texttt{ratio}\langle\texttt{A},\texttt{B}\rangle,\texttt{ratio}\langle\texttt{C},\texttt{D}\rangle\rangle} to be \texttt{false} in some cases. Some expressions of this form would become \texttt{true} under the present proposal and thus would likely have failed such an academic, implementation-oriented test.}

3 Proposed wording\footnote{All proposed \textit{additions} and \textit{deletions} are relative to the post-Kona Working Draft [N4659]. Editorial notes are displayed against a gray background.}

3.1 Edit [ratio.ratio] (23.16.3) as shown:

\begin{verbatim}
namespace std {
    template <intmax_t N, intmax_t D = 1>
    class ratio Ratio {  // exposition only
        public:
            static constexpr intmax_t num;  // see below
            static constexpr intmax_t den;  // see below
            using type = ratio Ratio<num, den>;
        };
    template <intmax_t N, intmax_t D = 1>
    using ratio = typename Ratio<N, D>::type;  // alias the reduced form
}
\end{verbatim}

1 If the program is ill-formed if (a) the template argument \( D \) is zero or (b) the absolute values of either of the template arguments \( N \) and \( D \) is not representable by type \texttt{intmax\_t}, the program is ill-formed. \textit{[Note: These rules ensure (a) that infinite ratios are avoided and (b) that for any negative input, there exists a representable value of its absolute value which is positive. In a two’s complement representation, this excludes the most negative value. — end note]}

2 The static data members \( \texttt{num} \) and \( \texttt{den} \) shall have the following values, where \( \texttt{gcd} \) represents the greatest common divisor of the absolute values of \( N \) and \( D \):

\begin{equation}
\text{num} \text{ shall have the value } \text{sign}(N) \ast \text{sign}(D) \ast \text{abs}(N) \ast \text{gcd}.
\end{equation}
(2.2) — *den* shall have the value \( \frac{\text{abs}(D)}{\text{gcd}} \).

3 [Example: Each of the following expressions is true.]

- \( \text{is\_same\_v<ratio<5,10>, ratio<1,2>>} \)
- \( \text{is\_same\_v<ratio<5,10>, ratio<2,4>>} \)
- \( \text{is\_same\_v<ratio<2,4>, ratio<1,2>>} \)

— end example —

3.2 Edit [ratio.arithmetic] (23.16.4) as shown:

1 Each of the alias templates *ratio_add*, *ratio_subtract*, *ratio_multiply*, and *ratio_divide* denotes the result of an arithmetic computation on two ratios \( R_1 \) and \( R_2 \). With \( \mathbf{X}_N \) and \( \mathbf{Y}_D \) computed (in the absence of arithmetic overflow) as specified by Table 51, each alias denotes a ratio \( \mathbf{U}, \mathbf{V} \) such that \( \mathbf{U} \) is the same as \( \text{ratio}<\mathbf{X}, \mathbf{Y}>::\text{num} \) and \( \mathbf{V} \) is the same as \( \text{ratio}<\mathbf{X}, \mathbf{Y}>::\text{den} \).

2 If it is not possible to represent \( \mathbf{U}_N \) or \( \mathbf{V}_D \) with \text{intmax}_t, the program is ill-formed. Otherwise, an implementation should yield correct values of \( \mathbf{U}_N \) and \( \mathbf{V}_D \). If it is not possible to represent \( \mathbf{X} \) or all intermediate results with \text{intmax}_t, the program is ill-formed unless the implementation nonetheless yields correct values of \( \mathbf{U}_N \) and \( \mathbf{V}_D \).

3.3 Edit column headings of Table 51 as shown:

<table>
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<tr>
<th>Type</th>
<th>Value of ( \mathbf{X}_N )</th>
<th>Value of ( \mathbf{Y}_D )</th>
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4 An alternative

We also considered a different approach that does not alter *ratio*’s specification, but introduces the following new alias family instead:

```
1 template< intmax_t N, intmax_t D = 1 >
2 using
3   ratio_t = typename ratio<N, D>::type;  // reduced form of ratio<N,D>
```

A different name is, of course, possible; \text{reduced\_ratio}_t has already been suggested.

While we do not object to introducing such a new typedef-name, it does require users to be explicit about when they want a *ratio* in its reduced form. Based on our test results from the first approach, we find no cognitive, technical, or computational benefit in maintaining any unreduced *ratio*, nor in requiring users to specify when a reduced *ratio* is wanted.

5 Acknowledgments

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6 Bibliography


7 Document history

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