Abstract

This paper proposes to add the following type of classes with the associated customization points and some algorithms that work well with them.

- **Functor**
- **Applicative**
- **Monad**
- **Monad-Error**

This paper concentrates on the basic operations. More will come later if the committee accept the design (See Future Work section).

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History

Revision 1

This is a minor revision

- Adapt to new `std::unexpected` interface as for P0323R3.
- Get rid of `xxx::tag` to detect the concept.
Revision 0

Creation in response to request of the committee to split the expected proposal [P0323R0](#) into an expected class [P0323R0](#) and a monadic interface (this document).

Introduction

Most of you know *Functor, Applicative, Monad* and *MonadError* from functional programming. The underlying types of the types modeling *Functor, Applicatives, Monad* and *MonadError* are homogeneous, that is, the functions have a single type.

In the following notation \([T]\) stands for a type wrapping instances of a type \(T\), possibly zero or \(N\) instances. \((T \to U)\) stands for a function taking a \(T\) as parameter and returning a \(U\).

Next follows the signatures proposed by this paper.

```plaintext
functor::transform : [T] x (T->U) -> [U]
functor::map : (T->U) x [T] -> [U]
applicative::ap : [T] x [(T->U)] -> [U]
applicative::pure<A> : T -> [T]
monad::unit<A> : T -> [T]
monad::bind : [T] x (T->[U]) -> [U] //mbind
monad::unwrap : [[T]] -> [T] //unwrap
monad::compose : (B->[C]) x (A->[B])-> (A->[C])
monad_error::make_error<M>: E -> error_type_t<M,E>
monad_error::catch_error: [T] x (E->T) -> [T] where E = error_type_t<[T]>
monad_error::catch_error: [T] x (E->[T]) -> [T]
```

Motivation and Scope

From Expected proposals

Adapted from [P0323R0](#) taking in account the proposed non-member interface.

Safe division

This example shows how to define a safe divide operation checking for divide-by-zero conditions. Using exceptions, we might write something like this:
```cpp
struct DivideByZero: public std::exception {...};

double safe_divide(double i, double j)
{
    if (j==0) throw DivideByZero();
    else return i / j;
}
```

With `expected<T,E>`, we are not required to use exceptions, we can use `std::error_condition` which is easier to introspect than `std::exception_ptr` if we want to use the error. For the purpose of this example, we use the following enumeration (the boilerplate code concerning `std::error_condition` is not shown):

```cpp
enum class arithmetic_errc
{
    divide_by_zero, // 9/0 == ?
    not_integer_division // 5/2 == 2.5 (which is not an integer)
};
```

Using `expected<double, error_condition>`, the code becomes:

```cpp
expected<double, error_condition> safe_divide(double i, double j)
{
    if (j==0) return unexpected(arithmetic_errc::divide_by_zero); // (1)
    else return i / j; // (2)
}
```

(1) The implicit conversion from `unexpected<E>` to `expected<T,E>` and (2) from `T` to `expected<T,E>` prevents using too much boilerplate code. The advantages are that we have a clean way to fail without using the exception machinery, and we can give precise information about why it failed as well. The liability is that this function is going to be tedious to use. For instance, the exception-based

```cpp
function i + j/k is:

double f1(double i, double j, double k)
{
    return i + safe_divide(j,k);
}
```

but becomes using `expected<double, error_condition>`:

```cpp
expected<double, error_condition> f1(double i, double j, double k)
{
    auto q = safe_divide(j, k)
    if (q) return i + *q;
    else return q;
}
```

This example clearly doesn't respect the “clean code” characteristic and the readability doesn't differ much from the “C return code”. Hopefully, we can see `expected<T,E>` through functional glasses as a monad. The code is cleaner using the function `functor::transform`. This way, the error handling is not explicitly mentioned but we still know, thanks to the call to `transform`, that something is going underneath and thus it is not as silent as exception.
```cpp
expected<double, error_condition> f1(double i, double j, double k) {
    return functor::transform(safe_divide(j, k), [&](double q) {
        return i + q;
    });
}
```

The `transform` function calls the callable provided if `expected` contains a value, otherwise it forwards the error to the callee. Using lambda function might clutter the code, so here the same example using functor:

```cpp
expected<double, error_condition> f1(double i, double j, double k) {
    return functor::transform(safe_divide(j, k), bind(plus, i, _1));
}
```

We can use `expected<T, E>` to represent different error conditions. For instance, with integer division, we might want to fail if the two numbers are not evenly divisible as well as checking for division by zero. We can overload our `safe_divide` function accordingly:

```cpp
expected<int, error_condition> safe_divide(int i, int j) {
    if (j == 0) return unexpected(arithmetic_errc::divide_by_zero);
    if (i%j != 0) return unexpected(arithmetic_errc::not_integer_division);
    else return i / j;
}
```

Now we have a division function for integers that possibly fail in two ways. We continue with the exception oriented

```cpp
//function i/k + j/k:
int f2(int i, int j, int k) {
    return safe_divide(i,k) + safe_divide(j,k);
}
```

Now let’s write this code using an `expected<T, E>` type and the functional `transform` already used previously.

```cpp
expected<int, error_condition> f(int i, int j, int k) {
    return monad::bind(safe_divide(i, k), [=](int q1) {
        return functor::transform(safe_divide(j, k), [=](int q2) {
            return q1+q2;
        });
    });
}
```

The compiler will gently say he can convert an `expected<expected<int, error_condition>, error_condition>` to `expected<int, error_condition>` . This is because the function `functor::transform` wraps the result in `expected` and since we use twice the map member it wraps it twice. The function `monad::bind` (do not confound with `std::bind`) wraps the result of the continuation only if it is not already wrapped. The correct version is as follow:
The error-handling code has completely disappeared but the lambda functions are a new source of noise, and this is even more important with `expected` variables. Propositions for a better monadic experience are discussed in section [Do-Notation], the subject is left open and is considered out of scope of this proposal.

### Error retrieval and correction

The major advantage of `expected<T,E>` over `optional<T>` is the ability to transport an error, but we didn’t come yet to an example that retrieve the error. First of all, we should wonder what a programmer do when a function call returns an error:

1. Ignore it.
2. Delegate the responsibility of error handling to higher layer.
3. Trying to resolve the error.

Because the first behavior might lead to buggy application, we won’t consider it in a first time. The handling is dependent of the underlying error type, we consider the `exception_ptr` and the `error_condition` types.

We spoke about how to use the value contained in the `expected` but didn’t discuss yet the error usage.

A first imperative way to use our error is to simply extract it from the `expected` using the `error()` member function. The following example shows a `divide2` function that return 0 if the error is `divide_by_zero`:

```cpp
expected<int, error_condition> divide2(int i, int j)
{
    auto e = safe_divide(i, j);
    if (!e && e.error().value() == arithmetic_errc::divide_by_zero) {
        return 0;
    }
    return e;
}
```

This imperative way is not entirely satisfactory since it suffers from the same disadvantages than `value()`. Again, a functional view leads to a better solution. The `catch_error` member calls the continuation passed as argument if the expected is erroneous.
expected `int, error_condition` divide3(`int i, int j`)
{
    auto e = safe_divide(i, j);
    return monad_error::catch_error(e, [] (const error_condition & e) {
        if (e.value() == arithmetic_errc::divide_by_zero)
        {
            return 0;
        }
        return unexpected(e);
    });
}

An advantage of this version is to be coherent with the `monad::bind` and `functor::map` functions. It also provides a more uniform way to analyze error and recover from some of these. Finally, it encourages the user to code its own “error-resolver” function and leads to a code with distinct treatment layers.

**Proposal**

This paper proposes to add the following type of classes with the associated customization points and the algorithms that work well with them.

- **Functor**
- **Applicative**
- **Monad**
- **Monad-Error**

These are the basic operations. More will come later if the committee adopt the design (See Future Work section).

**Design Rationale**

Most of the design problems for this library are related to the names, signatures and how this type of classes are customized. See [CUSTOM](#) for a description of an alternative approach to customization points. This proposal is based on this alternative approach, but it could be adapted to other approaches once we decide which is the mechanism we want to use.

**Functor**

**functor::transform** versus **functor::map**

The signature of the more C++ `transform` function is different from the usual `Functor map` function.

\[
\begin{align*}
\text{transform} & : [T] \times (T \to U) \to [U] \\
\text{map} & : (T \to U) \times [T] \to [U]
\end{align*}
\]

`transform` has the advantage to be closer to the STL signature.

The advantage of the `map` is that it can be extended to a variadic number of `Functors`.

\[
\text{map} : (T_1 \times \ldots \times T_n \to U) \times [T_1] \times \ldots \times [T_n] \to [U]
\]
Both seem to be useful, and so we propose both in this paper.

**Applicative**

```cpp
applicative::ap
```

TODO Add some motivation and rationale for `ap`.

```cpp
applicative::pure
```

We don't define an additional `applicative::pure` function as we have already `type_constructuble::make`.[P0338R2]

**Monad**

```cpp
monad::unit
```

We don't define an additional `monad::unit` function as we have already `type_constructuble::make`.[P0338R2]

```cpp
monad::bind
```

C++ has the advantage to be able to overload on the parameters of the signature.

```cpp
bind
```

can be overloaded with functions that return a `Monad` or functions that return the `ValueType` as it proposed for `std::experimental::future::then`.

The authors don't see any inconvenient in this overload, but would prefer to have an additional function that supports this ability, so that we know that chain will only work with functions that return a `Monad`.

Note that the user could use `transform` and `bind` to get this overload.

```cpp
monad::bind  function parameter parameter
```

The `bind` function accepts functions that take the `ValueType` as parameter. `std::experimental::future::then` function parameter takes a `future<T>`.

```cpp
monad::unwrap
```

This is an alternative way to define a `Monad`.

We define it in function of `monad::bind` and define `monad::bind` in function of `monad::unwrap`.

```cpp
monad::compose
```

This is the composition of monadic functions.

**Customization**

**ADL versus traits**

These concepts have some functions that cannot be customized using overload (ADL), as the dispatching type is not a function.
parameters, e.g. `pure<TC>(C)` or `make_error<TC>(E)`.

We have also some customization points that are types, as `error_type<T>::type`.

The authors prefer to have a single customization mechanism, and traits is the one that allows to do everything.

**Boost.Hana** uses a similar mechanism.

See [CUSTOM](#) where we describe the advantages and liabilities of each approach.

### All at once or one by one

**Boost.Hana** has chosen to customize each operation individually. The approach of this proposal is closer to how other languages have addressed the problem, that is, customize all operations at once.

There are advantages and liabilities to both approaches. See [CUSTOM](#) where we describe the advantages and liabilities of each approach.

### Allow alternative way to customize a type of classes

Some type of classes can be customized using different customization points. E.g. *Monad* can be customized by either defining *bind* or *unwrap*. The other customization points being defined in function of others.

This proposal uses an emulation to what Haskel calls minimal complete definition, that is a struct that defines some operations given the user has customized the minimal ones.

### About names

There is a tradition in functional languages as Haskell with names that could be not the more appropriated for C++.

**functor::map** alternatives

We have already a clear meaning of `map` in the standard library, the associative ordered container `std::map`. The proposed `functor::map` function is in scope `std::experimental::functor::map` so there is no possible confusion. Haskell uses `fmap` instead of `functor::map` as it has no namespaces, but we have them in C++. **Boost.Hana** doesn't provides it.

**applicative::pure** versus **type_constructible::make**

Haskell uses `pure` as factory of an applicative functor. The standard library uses `make_` for factories. In addition we have already the proposed `type_constructible::make` [P0338R2](#) that plays the same role.

**Boost.Hana** uses `lift`. However **Boost.Hana** provides also a Core `make` facility not associated to any concept.

**applicative::ap** versus **applicative::apply**

**monad::unit** versus **type_constructible::make**

**monad::bind** versus **monad::chain**

We have already a clear meaning of `bind` in the standard library function `std::bind`, which could be deprecated in a future as we have now lambda expressions. The proposed `bind` (Haskell uses `mbind`) is in scope `std::experimental::monad::bind` so there is no possible confusion. **Boost.Hana** uses `chain` instead. **Boost.Hana**
locates all the function isn namespace boost::hana.

We could define `bind` in function of a possibly `then` function (or whatever is the appropriated name) when the type provides access to the `ValueType` as it is the case for `std::future` and all the `ValueOrError` types P0786R0. However the authors don't know how to do it in the general case.

-monad::unwrap versus monad::flatten versus monad::join-

[THEN] original proposal had a future::unwrap function that unwraps a wrapped future. Haskell uses join. Boost.Hana uses flatten.

-monad_error::throw_error versus monad_error::make_error-

Haskell uses throw_error as factory for monaderror errors. If we choose make to wrap a value, it seems coherent to use make_error instead of throwerror as C++ has exceptions. We are not throwing an error but building it. We have the proposed type_constructible::make P0338R2 that plays the same role.

Operators

operators namespace

Given that C++ has not the possibility to add new operators with a specific precedence and associativity, is is difficult to reuse one of the current operators to make the map or bind operations more friendly. We need a left associative operator. Some have tried with operator|() for functor::map and operator>= for monad::bind. However these operator have no the precedence we would need and the user would need to use parenthesis more often than expected.

The authors consider that this is good for playing with the concepts, but are not good for the C++ standard.

Language based syntactic sugar

Haskell is a functional language where everything is an expression. It has a specific syntax sugar for the monadic bind operation. It is the do-notation, which makes the Haskell language to look more like an imperative language.

We have already the operator co_await in the Coroutine TS. While this works well for a lot of Monads, it doesn't works well for Monads representing non-determinism.

There is another ongoing proposal for operator try applicable to ValueOrError types, but this doesn't covers all the Monads neither.

We could have a proposal to include some kind of do-notation that is more adapted to the C++ language, or adapt the operator co_await to take care of non-determinism. The authors don't know how to do it yet.

Customization

This paper is based on an alternative customization approach CUSTOM. While this is not an imperative this approach helps to define such concepts.

Factory functions

Both Applicative and Monad have factory function applicative::pure and monad::unit. We have already such a factory function isolated in the TypeConstructible concept via type_constructible::make.
We could define those specific factory functions but functions that forward to the `factory::make` function, but there is not too much interest in duplicating such factories. We can just nest the factory namespace in `applicative` meaning that any `Applicative` is a `TypeConstructible`.

**Impact on the standard**

These changes are entirely based on library extensions and do not require any language features beyond what is available in C++17. There are however some classes in the standard that need to be customized.

This paper depends in some way on the helper classes proposed in P0343R0, as e.g. the place holder `_t` and the associated specialization for the type constructors `optional<_t>`, `unique_ptr<_t>`, `shared_ptr<_t>`.

**Proposed Wording**

The proposed changes are expressed as edits to N4617 the Working Draft - C++ Extensions for Library Fundamentals V2.

Add a "Functor Types" section

**Functor Types**

**Functor requirements**

A **Functor** is a type constructor that supports the `transform` function. A type constructor `TC` meets the requirements of **Functor** if:

- `TC` is a `TypeConstructor`
- for any `T` `EqualityComparable DefaultConstructible`, and `Destructible`, `invoke_t<TC,T>` satisfies the requirements of `EqualityComparable DefaultConstructible`, and `Destructible`,
- the expressions shown in the table below are valid and have the indicated semantics, and
- `TC` satisfies all the other requirements of this sub-clause.

In Table X below, `t` denotes an rvalue of type `invoke<TC,T>`, `f` denotes a rvalue of type `F` where `F` satisfies `Callable`.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Return Type</th>
<th>Operational Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>invoke_t&lt;TC, VT...&gt;</code></td>
<td>T</td>
<td></td>
</tr>
<tr>
<td><code>type_constructor_t&lt;T&gt;</code></td>
<td>TC</td>
<td></td>
</tr>
<tr>
<td><code>functor::transform(t, f)</code></td>
<td><code>invoke_t&lt;TC,U&gt;</code></td>
<td>Applies <code>f</code> to the contents of <code>t</code> and wraps the result with the functor. Equivalent to <code>functor::adjust_if(x, always(true), f)</code></td>
</tr>
<tr>
<td><code>functor::adjust_if(t, p, f)</code></td>
<td><code>invoke_t&lt;TC,U&gt;</code></td>
<td>Applies <code>f</code> to the contents of <code>t</code> if the predicate <code>p</code> applied to the contents of <code>t</code> is <code>true</code> or just the contents of <code>t</code>; then wraps the previous result with the functor. Equivalent to <code>functor::transform(x, [&amp;](auto x) { if pred(x) return f(x) else return x; })</code></td>
</tr>
</tbody>
</table>

**Header synopsis [functor.synop]**
namespace std {
namespace experimental {
inline namespace fundamentals_v3 {
namespace functor {

// class traits
template <class TC, class Enabler=void>
struct traits {};

template <class T, class F>
'see below' transform(T&& x, F&& f);
template <class T, class P, class F>
'see below' adjust_if(T&& x, P&& p, F&& f);

struct mcd_transform;
struct mcd_adjust_if;
}

template <class T>
struct is_functor;
template <class T>
inline constexpr bool is_functor_v = is_functor<T>::value;
}

namespace functor {
    struct is_functor;
    template <class T>
    struct is_functor<const T> : is_functor<T> {};
    template <class T>
    struct is_functor<volatile T> : is_functor<T> {};
    template <class T>
    struct is_functor<const volatile T> : is_functor<T> {};
}
}

Class Template traits [functor.traits]

namespace functor {
    template <class T, class Enabler=void>
    struct traits {}; 
}

Remark The Enabler parameter is a way to allow conditional specializations.

Function Template transform [functor.transform]

namespace functor {
    template <class T, class F>
    auto transform(T&& x, F&& f)
}

Let TC be type_constructor<decay_t<T>>

Effects: forward the call to the traits<TC>::transform
Remark: The previous function shall not participate in overload resolution unless:

- T has a type constructor TC that satisfies Functor,
- F is a Callable taking as parameter the ValueType of T and result U,
- The result of transform is the rebinding of T with the result of the invocation of f with the value of x.

transform : [T] x T->U -> [U]

**Function Template adjust_if [functor.adjust_if]**

```cpp
namespace functor {
    template <class T, class P, class F>
    auto adjust_if(T&& x, P&& p, F&& f);
}
```

Let TC be type_constructor<decay_t<T>>

**Effects:** forward the call to the traits<TC>::adjust_if

Remark: The previous function shall not participate in overload resolution unless:

- T has a type constructor TC that satisfies Functor,
- F is a Callable taking as parameter the ValueType of T and result U,
- P is a Predicate taking as parameter the ValueType of T,
- The result of adjust_if is the rebinding of T with the result of the invocation of f with the value of x.

adjust_if : [T] x T->bool x T->U -> [U]

**class mcd_transform [functor.mcd_transform]**

```cpp
namespace functor {
    struct mcd_transform {
        template <class T, class P, class F>
        auto adjust_if(T&& x, P&& p, F&& f);
    };
}
```

This minimal complete definition defines adjust_if in function of transform.

**class mcd_transform:: adjust_if [functor.mcd_transform.adjust_if]**

```cpp
namespace functor {
    template <class T, class P, class F>
    auto mcd_transform::adjust_if(T&& x, P&& p, F&& f);
}
```

Equivalent to:
return functor::transform(x, [&](auto x) { if pred(x) return f(x) else return x; });

class mcd_adjust_if [functor.mcdadjustif]

namespace functor {
    struct mcd_adjust_if {
        template <class T, class F>
        auto transform(T&& x, F&& f);
    }
}

This minimal complete definition define transform in function of adjust_if.

class mcd_adjust_if::transform [functor.mcdadjustif.transform]

namespace functor {
    template <class T, class F>
    auto mcd_adjust_if::transform(T&& x, F&& f);
}

Equivalent to:

return functor::adjust_if(x, always(true), f);

where always(true) is a function object that return always true.

Template class is_functor [functor.is_functor]

template <class T>
struct is_functor;

Add a "Applicative Types" section

Applicative Functor Types

Applicative requirements

A Applicative is a type constructor that supports the Functor requirements, the TypeConstructible requirements and supports the \texttt{ap} function.

A type constructor \texttt{TC} meets the requirements of Applicative if:

\begin{itemize}
  \item \texttt{TC} is a \texttt{Functor} and \texttt{TypeConstructible},
  \item the expressions shown in the table below are valid and have the indicated semantics, and
  \item \texttt{TC} satisfies all the other requirements of this sub-clause.
\end{itemize}

In Table X below, \(a\) denotes an rvalue of type \texttt{\texttt{invoke<TC,T>}}, \(f\) denotes a rvalue of type \texttt{\texttt{invoke<TC,T>}} where \(F\) satisfies Callable.
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</tr>
<tr>
<td>type_constructor_t&lt;T&gt;</td>
<td>TC</td>
<td></td>
</tr>
<tr>
<td>applicative::ap(a, f)</td>
<td>rebind_t&lt;TC,U&gt;</td>
<td>Applies the contents of <code>f</code> to the contents of <code>a</code>.</td>
</tr>
</tbody>
</table>

**Header synopsis [functor.synop]**

```cpp
namespace std {
namespace experimental {
inline namespace fundamentals_v3 {
namespace applicative {
    using namespace functor;

    // class traits
    template <class TC, class Enabler=void>
    struct traits {};

    template <class A, class F>
    `see below` ap(A&& x, F&& f);
}

    template <class T>
    struct is_applicative;
    template <class T>
    inline constexpr bool is_applicative_v = is_applicative<T>::value;
    template <class T>
    struct is_applicative<const T> : is_applicative<T> {};
    template <class T>
    struct is_applicative<volatile T> : is_applicative<T> {};
    template <class T>
    struct is_applicative<const volatile T> : is_applicative<T> {};
}
}
}
```

**Class Template traits [functor.traits]**

```cpp
namespace functor {
    template <class T, class Enabler=void>
    struct traits {};
}
```

*Remark* The Enabler parameter is a way to allow conditional specializations.

**Function Template ap [applicative.ap]**

```cpp
namespace applicative {
    template <class A, class F>
    auto ap(A&& x, F&& f)
}
```
Let $TC$ be $\text{type\_constructor<decay\_t<A>>}$

**Effects:** forward the call to the $\text{traits<TC>::ap}$.

**Remark:** The previous function shall not participate in overload resolution unless:

- $A$ has a type constructor $TC$ that satisfies $\text{Applicative}$,
- $F$ has a type constructor $TC$ that satisfies $\text{Applicative}$,
- $\text{value\_type_t<F>\_t<U>}$ is a $\text{Callable}$ taking as parameter the $\text{ValueType}$ of $T$ and result $U$,
- The result of $\text{ap}$ is the rebinding of $T$ with the result of the invocation of the contents of $f$ with the value of $x$.

\[
ap : [T] \times [T\rightarrow U] \rightarrow [U]\]

### Template class is_applicative [applicative.is_applicative]

```cpp
template <class T>
struct is_applicative;
```

Add a "Monad Types" section

## Monad Types

### Monad requirements

A Monad is a type constructor that in addition to supporting $\text{Applicative}$ supports the $\text{bind}$ function. A type constructor $TC$ meets the requirements of Monad if:

- $TC$ is a $\text{TypeConstructor}$
- for any $T$ $\text{EqualityComparable}$ $\text{DefaultConstructible}$, and $\text{Destructible}$, $\text{invoke\_t<TC,T>}$ satisfies the requirements of $\text{EqualityComparable}$ $\text{DefaultConstructible}$, and $\text{Destructible}$,
- the expressions shown in the table below are valid and have the indicated semantics, and
- $TC$ satisfies all the other requirements of this sub-clause.

In Table X below, $m$ denotes an rvalue of type $\text{invoke<TC,T>}$, $f$ denotes a $\text{Callable}$ rvalue of type $F$. In Table X below, $m$ denotes an rvalue of type $\text{invoke<TC,T>}$, $f$ denotes a rvalue of type $F$ where $F$ satisfies $\text{Callable(T)}$.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Return Type</th>
<th>Operational Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{invoke_t&lt;TC,T,V...&gt;}$</td>
<td>$T$</td>
<td></td>
</tr>
<tr>
<td>$\text{type_constructor_t&lt;T&gt;}$</td>
<td>$TC$</td>
<td></td>
</tr>
<tr>
<td>$\text{monad::bind(m,f)}$</td>
<td>$\text{invoke_t&lt;TC,U&gt;}$</td>
<td>Applies <code>f</code> to the contents of <code>m</code> if any.</td>
</tr>
<tr>
<td>$\text{monad::unwrap(nm)}$</td>
<td>$\text{invoke_t&lt;TC,T&gt;}$</td>
<td>Extract the contents of <code>nm</code> if any.</td>
</tr>
</tbody>
</table>

### Header synopsis [monad.synop]
namespace std {
namespace experimental {
inline namespace fundamentals_v3 {
namespace monad {
    using namespace applicative;

    // class traits
    template <class TC, class Enabler=void>
    struct traits {};

template <class T, class F>
    'see below' bind(T&& x, F&& f);

template <class T>
    'see below' unwrap(T&& x);

    struct mcd_bind;
    struct mcd_unwrap;
}

    template <class T>
    struct is_monad;
    template <class T>
        inline constexpr bool is_monad_v = is_monad<T>::value;

    template <class T>
        struct is_monad<const T> : is_monad<T> {};
    template <class T>
        struct is_monad<volatile T> : is_monad<T> {};
    template <class T>
        struct is_monad<const volatile T> : is_monad<T> {};
}
}
}

Class Template traits [monad.traits]

namespace monad {
    template <class T, class Enabler=void>
        struct traits {};
}

Remark The Enabler parameter is a way to allow conditional specializations.

Function Template transform [monad.bind]

namespace monad {
    template <class M, class F>
        auto bind(M&& x, F&& f)
}

Let TC be type_constructor<decay_t<M>>
Let T be value_type<decay_t<M>>
Effects: forward the call to the `traits<TC>::bind` function. This function must return the result of calling to the `f` parameter with the contained value type, if any; Otherwise it must return a monad of the same type that `F` returns without a value type.

Remark: The previous function shall not participate in overload resolution unless:

- `M` has a type constructor `TC` that satisfies `Monad`,
- `F` satisfies `Callable<F, invoke_t<TC,U>(T)>` where `T` is the `ValueType` of `M` for some type `U`,
- The result of `bind` is the result of the invocation of `f` with the value of `x` if any, otherwise an `invoke_t<TC,U>(T)` instance without a value.

```plaintext
bind : [T] x T->[U] -> [U]
```

Function Template `unwrap [monad.unwrap]`

```plaintext
namespace monad {
  template <class M>
    auto unwrap(M&& x)
}
```

Let `TC` be `type_constructor<decay_t<M>>`

Effects: forward the call to the `traits<TC>::unwrap` function. This function should flatten input `Monad` on a `Monad` that has one less nested level.

Remark: The previous function shall not participate in overload resolution unless:

- `M` has a type constructor `TC` that satisfies `Monad`,
- `M` has the form `TC<TC<T>>` where `T` is `value_type_t<value_type_t<decay_t<M>>>`
- The result of `unwrap` is the monad `TC<T>`.

```plaintext
unwrap : [[T]] -> [T]
```

Class `mcd_bind [monad.mcd_bind]`

```plaintext
namespace monad {
  struct mcd_bind
  {
    template <class T>
      auto unwrap(T&& x);
  }
}
```

This minimal complete definition define `unwrap` in function of `bind`.

Class `mcd_bind::unwrap [monad.mcd_bind.unwrap]`

```plaintext
namespace monad {
  template <class T>
    auto mcd_bind::unwrap(T&& x);
}
```
Equivalent to:

```cpp
monad::bind(x, identity, f);
```

where `identity` is a unary function object that return its parameter.

**Class `mcd_unwrap` [monad.mcd_unwrap]**

```cpp
namespace monad {
    struct mcd_unwrap {
        template <class T, class F>
        auto bind(T&& x, F&& f);  
    };
}
```

This minimal complete definition defines `bind` in function of `unwrap` and `transform`.

**Class `mcd_unwrap::bind` [monad.mcd_unwrap.bind]**

```cpp
namespace monad {
    template <class T, class F>
    auto mcd_unwrap(T&& x, F&& f);
}
```

Equivalent to:

```cpp
monad::unwrap(functor::transform(x, f));
```

**Template class `is_monad` [monad.is_monad]**

```cpp
template <typename T>
struct is_monad;
```

Add a "Monad Error Types" section

**Monad Error Types**

**MonadError requirements**

A `MonadError` is a type constructor that in addition to supporting `Monad` supports the `make_error` and the `catch_error` functions. A type constructor `TC` meets the requirements of `MonadError` if:

- `TC` is an `Monad`
- the expressions shown in the table below are valid and have the indicated semantics, and
- `TC` satisfies all the other requirements of this sub-clause.

In Table X below, `m` denotes an rvalue of type `invoke<TC,T>`, `f` denotes a `Callable` rvalue of type `F`. In Table X below,
\( m \) denotes an rvalue of type \( \text{invoke}<T, T> \), \( f \) denotes a rvalue of type \( F \) where \( F \) satisfies \( \text{Callable}(T) \).

<table>
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<tr>
<th>Expression</th>
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<tbody>
<tr>
<td>( \text{invoke}_t&lt;T, VT...&gt; )</td>
<td>( T )</td>
<td></td>
</tr>
<tr>
<td>( \text{type_constructor}_t&lt;T&gt; )</td>
<td>( TC )</td>
<td></td>
</tr>
<tr>
<td>( \text{error_type}_t&lt;TC&gt; )</td>
<td>( E )</td>
<td></td>
</tr>
<tr>
<td>( \text{monad}_\text{error}::\text{make}_\text{error}(e) )</td>
<td>( \text{Err} )</td>
<td>a instance of a type depending on ( \text{error_type}_t&lt;TC&gt; ) that is convertible to any ( \text{invoke}_t ).</td>
</tr>
<tr>
<td>( \text{monad}_\text{error}::\text{catch}_\text{error}(m, f) )</td>
<td>( M )</td>
<td>Applies ( f ) to the error of ( m ) if any. Otherwise it return ( m ).</td>
</tr>
</tbody>
</table>

Header synopsis [\texttt{monad\_error.synop}]

```cpp
namespace std {
namespace experimental {
inline namespace fundamentals_v3 {
namespace monad_error {
  using namespace monad;

  // class traits
  template <class TC, class Enabler=void>
  struct traits {};

  template <class M>
  struct error_type
  {
    using type = typename traits<M>::template error_type<M>;
  };

  template <class M>
  using error_type_t = typename error_type<M>::type;

  template <class T, class F>
  'see below' catch_error(T&& x, F&& f);
}

  template <class T>
  struct is_monad_error;
}
```

```cpp
  template <class T>
  inline constexpr bool is_monad_error_v = is_monad_error<T>::value;

  template <class T>
  struct is_monad_error<const T> : is_monad_error<T> {};

  template <class T>
  struct is_monad_error<volatile T> : is_monad_error<T> {};

  template <class T>
  struct is_monad_error<const volatile T> : is_monad_error<T> {};
```
**Class Template** traits [monad_error.traits]

```cpp
namespace monad_error {
    template <class T, class Enabler=void>
    struct traits {};
}
```

*Remark* The `Enabler` parameter is a way to allow conditional specializations.

**Function Template** catch_error [monad_error.catcherror]

```cpp
namespace monad_error {
    template <class M, class F>
    auto catch_error(M&& x, F&& f)
}
```

Let $TC$ be `type_constructor<decay_t<M>>`

Let $T$ be `value_type<decay_t<M>>`

Let $E$ be `error_type<decay_t<M>>`

*Effects:* forward the call to the `traits<TC>::catch_error`. This function must return the result of calling to the `f` parameter with the contained error type, if any; Otherwise it must returns the parameter `x`.

*Remark:* The previous function shall not participate in overload resolution unless:

- $M$ has a type constructor $TC$ that satisfies `monad`,
- $F$ satisfies `Callable<F, M(E)>` where $E$ is the `ErrorType` of $M$,
- The result of `catch_error` is the result of the invocation of `f` with the error of `x` if any, otherwise `x`.

`catch_error : [T]:E x E->[T] -> [T]:E`

**Function Template** recover [monad_error.recover]

```cpp
namespace monad_error {
    template <class M, class F>
    auto recover(M&& x, F&& f)
}
```

Let $TC$ be `type_constructor<decay_t<M>>`

Let $T$ be `value_type<decay_t<M>>`

Let $E$ be `error_type<decay_t<M>>`

*Effects:* forward the call to the `traits<TC>::catch_error`. This function must return the result of calling to the `f` parameter with the contained error type, if any; Otherwise it must returns the parameter `x`.

*Remark:* The previous function shall not participate in overload resolution unless:

- $M$ has a type constructor $TC$ that satisfies `monad`,
- $F$ satisfies `Callable<F, T(E)>` where $E$ is the `ErrorType` of $M$ and $T$ is the value type of $M$,
- The result of `recover` is the result of the invocation of `f` with the error of `x` wrapped on a `M` if any, otherwise `x`.
recover : $[T] : E \times E \rightarrow T \rightarrow [T] : E$

**Function Template adapt_error** [monaderror.adapterror]

```cpp
namespace monad_error {
    template <class M, class F>
    auto adapt_error(M&& x, F&& f)
}
```

Let $\mathbf{TC}$ be $\text{type\_constructor}<\text{decay\_t}<M>>$

Let $\mathbf{T}$ be $\text{value\_type}<\text{decay\_t}<M>>$

Let $\mathbf{E}$ be $\text{error\_type}<\text{decay\_t}<M>>$

**Effects:** forward the call to the $\text{traits}<\mathbf{TC}>::\text{catch\_error}$.
This function must return the result of calling to the $f$ parameter
with the contained error type, if any; Otherwise it must returns
the parameter $x$.

**Remark:** The previous function shall not participate in overload resolution unless:

- $M$ has a type constructor $\mathbf{TC}$ that satisfies $\text{monad}$,
- $F$ satisfies $\text{Callable}<F, G(E)>$ where $E$ is the $\text{ErrorType}$ of $M$ and $G$ is another error type,
- The result of $\text{adapt\_error}$ is the result of the invocation of $f$ with the error of $x$ wrapped
  on a $\mathbf{TC}$ if any, otherwise the value wrapped with $\mathbf{TC}$.

catch_error : $[T] : E \times E \rightarrow G \rightarrow [T] : G$

**Template class is_monad_error** [monaderror.ismonad_error]

```cpp
    template <class T>
    struct is_monad_error;
```

**Customization for ValueOrError Types**

Add Specializations of $\text{ Functor, Applicative, Monad}$ and $\text{MonadError}$.

ValueOrError objects can be seen as $\text{Functor, Applicative and Monad}$. 
namespace value_or_error {
    struct as_functor {
        template <class T, class F>
        static constexpr auto transform(T&& x, F&& f) {
            return value_or_error::transform(forward<T>(x), forward<F>(f));
        }
    }

    struct as_applicative {
        template <class T, class F>
        static constexpr auto ap(F&& f, T&& x) {
            return value_or_error::ap(forward<F>(f), forward<T>(x));
        }
    }

    struct as_monad: monad::tag {
        template <class M, class F>
        static constexpr auto bind(M&& x, F&& f) {
            return value_or_error::bind(forward<M>(x), forward<F>(f));
        }
    }

    struct as_monad_error {
        template <class M, class F>
        static constexpr auto catch_error(M&& x, F&& f) {
            return value_or_error::catch_error(forward<M>(x), forward<F>(f));
        }
    }
}

namespace functor {
    template <class N>
    struct traits<N, meta::when<
        is_value_or_error<N>::value && is_type_constructible<N>::value
    >> : value_or_error::as_functor {};
}

namespace applicative {
    template <class N>
    struct traits<N, meta::when<
        is_value_or_error<N>::value && is_type_constructible<N>::value
    >> : value_or_error::as_applicative {};
}

namespace monad {
    template <class N>
    struct traits<N, meta::when<
        is_value_or_error<N>::value && is_type_constructible<N>::value
    >> : value_or_error::as_monad {};
}

namespace monad_error {
    template <class N>
    struct traits<N, meta::when<
        is_value_or_error<N>::value && is_type_constructible<N>::value
    >> : value_or_error::as_monad_error {};
}

Customization for Expected Objects
Add Specialization of `expected` [expected.object.monadic_spec].

```cpp
namespace functor {
    template <class T, class E>
    struct traits<expected<T, E>>
    {
        template <class Expected, class F>
            static constexpr auto transform(Expected&& x, F&& f);
    };
}

namespace applicative {
    template <class T, class E>
    struct traits<expected<T, E>>
    {
        template <class Expected, class F>
            static auto ap(F&& f, Expected&& x);
    };
}

namespace monad {
    template <class T, class E>
    struct traits<expected<T, E>>
    {
        template <class M, class F>
            static constexpr auto bind(M&& x, F&& f);
    };
}

namespace monad_error {
    template <class T, class E>
    struct traits<expected<T, E>>
    {
        template <class M>
            using error_type = typename M::error_type;

        template <class M, class ...Xs>
            static constexpr auto make_error(Xs&& ...xs);

        template <class M, class F>
            static constexpr auto catch_error(M&& x, F&& f);
    };
}
```

**Implementability**

This proposal can be implemented as pure library extension, without any language support, in C++17.

**Open points**

The authors would like to have an answer to the following points if there is any interest at all in this proposal:

- Do we want the proposed customization approach?
- Do we want separated proposals for each type class?
- Should a smart pointer (a pointer) be considered a `Functor`?
- Should a `ValueOrError` be considered a `MonadError`?
Should `std::array`, `std::vector` be considered a `Functor`?
Should a `Product Type` be considered a `Functor, Applicative, Monad` when all the elements have the same type?

**Future work**

**Add more algorithms**

Based on what [Boost.Hana](http://www.boost.org) provides already, extend the basic functionality with useful algorithms.

**Functor algorithms**

```
functor::adjust : [T] x CT x (T->U) -> [U]
functor::fill : [T] x U -> [U]
functor::replace_if : [T] x (T->bool) x T -> [T]
functor::replace : [T] x CT x T -> [T]
```

**Applicative algorithms**

```
applicative::lift : [T] x (T->bool) x (T->U) -> [U]
```

**Monad algorithms**

```
monad::then : [[T]] -> [T] // do
monad::next : [T] x ([T]->U) -> [U] // then
monad::next : [T] x ([T]->[U]) -> [U]
```

**Add `Functor`, `Applicative` and `Monad` on heterogeneous types**

The proposed `Functor`, `Applicative`, `Monad` are homogeneous, that is all the elements have the same type. However `ProductTypes` are heterogeneous and we can see them as something like heterogeneous `Functors`, `Applicatives` and `Monads`. The function applied could be either a `ProductType` of the corresponding functions (`N-Functor`, `N-Applicative`, `N-Monad`), or polymorphic functions (`P-Functor`, `P-Applicative`, `P-Monad`).

Do we need `N-Functor`, `N-Applicative`, `N-Monad` that support `Product Type`? Do we need `P-Functor`, `P-Applicative`, `P-Monad` that support `Product Type`?

**Add Transformers**

Monadic types don't compose very well when they are nested the ones on the others. We need some kind of transformer that facilitates their composition. See [Haskell Transformers](http://www.haskell.org/). 

**See how to add `Alternative` Haskel type class**

**Add `Monoids` and `MonadPlus` type classes**
**Add Foldable type classes**

**Acknowledgements**

Thanks to Louis for his work on the monadic interface of Boost.Hana.

Special thanks and recognition goes to Technical Center of Nokia - Lannion for supporting in part the production of this proposal.

**References**

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  http://boostorg.github.io/hana/index.html

- **N4617** N4617 - Working Draft, C++ Extensions for Library Fundamentals, Version 2 DTS
  

- **P0088R0** Variant: a type-safe union that is rarely invalid (v5)
  
  http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2015/p0088r0.pdf

- **P0323R0** A proposal to add a utility class to represent expected monad (Revision 2)
  
  http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2016/p0323r0.pdf

- **P0323R3** A proposal to add a utility class to represent expected monad (Revision 4)
  

- **P0338R2** C++ generic factories
  
  http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2017/p0338r2.pdf

- **P0343R0** - Meta-programming High-Order functions
  
  http://www.open-std.org/JTC1/SC22/WG21/docs/papers/2016/p0343r0.pdf

- **P0786R0** ValuedOrError and ValueOrNone types
  
  http://www.open-std.org/jtc1/sc22/wg21/docs/papers/2017/p0786r0.html

- **SUM_TYPE** Sum Types https://github.com/viboes/std-make/blob/master/doc/proposal/sum_type/SumType.md

- **CUSTOM** An Alternative approach to customization points
  

- [THEN]