Vector and Wavefront Policies

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1 Motivation

Vector parallelism is insufficiently supported by the current Parallelism TS (N4507). The Parallelism TS does offer the par_vec policy, and there is some interest in a variant that restricts execution to a single thread; the result of such a restriction is the unseq policy proposed in this paper. Alas, this policy, though it allows a vectorization (exploiting vector hardware), it is excessively permissive and fails to express the necessary requirements for an important set of vectorizable loops of practical interest. As defined in N4507, par_vec allows:

"The invocation of element access functions ... are permitted to execute in an unordered fashion in unspecified threads and unsequenced with respect to one another within each thread. [Note: this means that multiple function object invocations may be interleaved on a single thread. – end note]"

Merely constraining par_vec to a single thread still allows permissive interleaving that would give undefined semantics to loops in the aforementioned set.

Here is a short example that falls in the gap, using for_loop from P0075 with vector_execution_policy proposed in this paper:

```cpp
void binomial(int n, float y[]) {
  for_loop( vec, 0, n, [&](int i) {
    y[i] += y[i+1];
  });
}
```

The call to for_loop is equivalent, except with more relaxed sequencing, to:

```cpp
void binomial(int n, float y[]) {
  for( int i=0; i<n; ++i )
    y[i] += y[i+1];
}
```

The for_loop example cannot safely use unseq or par_vec instead of vec, because that would result in unsequenced reads and writes of the same element of y when n≥2. Subsequent sections show some more examples that require vec instead of unseq.

2 Changes since R0

- Changed formal specification of wavefront ordering to use a much simpler horizontal match formulation instead of labeling each evaluation with a LIFO context.
- Added ordered_update and its helper class ordered_update_t.
3 Execution policies for vectorization

3.1 Unsequenced and vector execution policies

This paper proposes adding two new execution policies to the Parallelism TS, assuming the adoption of P0075. These policies add support for execution with relaxed sequencing restricted to a single OS thread:

- An `unsequenced_execution_policy` class and constant `unseq` analogous to the other policy types and constants in the Parallelism TS, with sequencing semantics similar to `parallel_vector_execution_policy`, but limited to a single OS thread.

- A `vector_execution_policy` class and constant `vec` that is similar to the policy above, but guarantees stronger sequencing, compatible with classic work in the field of vectorization. This policy is restricted to the indexed-based loop templates proposed in P0075.

The first policy has strictly weaker sequencing guarantees than the second. The following lattice summarizes the strength of their guarantees relative to each other and existing policies, with the weakest guarantees at the top.\(^1\)

```
  +----------------+  +----------------+  +----------------+
  | par_vec        |  | unseq           |  | par             |
  +----------------+  |----------------+  +----------------+
        |              |  |                |
  +----------------+  |----------------+  +----------------+
        |     seq      |  |     vec        |
  +----------------+  |----------------+  +----------------+
```

No compiler extensions are necessary for correct implementation; since an implementation is free to implement any policy higher on the lattice via a policy lower on the lattice, serial execution is always allowed. The goal, however, is for the implementation to exploit parallel hardware, especially vector units, for improved performance. Some combination of OpenMP directives and vendor-specific hooks are likely to be used for implementing algorithms with either policy.\(^2\)

---

\(^1\)We also recommend that the existing `par_vec` be renamed `par_unseq` since the top lattice point’s relaxations are the union of the relaxations of `par` and `unseq`, or dually the top lattice point’s guarantees are the intersection of the guarantees of `par` and `unseq`.

\(^2\)In particular, we implemented a performant version of vector reductions for `for_loop` in LLVM by adding special intrinsics.
The ability to constrain execution to a single OS thread is commonly useful for avoiding resource interference with multi-threading designs.

Having two new policies, instead of one, and restricting vec to for_loop resolves a fundamental conflict. The unseq policy is generally useful and straightforward to define for the parallel algorithms in the Parallelism TS, but fails to capture guarantees critical to an important class of loops. Conversely, vec is critically useful for an important class of loops and definable for for_loop, but seems impractical to generalize to the parallel algorithms in a way that is both well-defined and beneficial to exploit.

3.2 Extensibility of Policies

Though we don’t propose it for standardization at this time, we note that vector_execution_policy could be subclassed to provide additional information from the programmer to the compiler. Providing this information as static const member of integral type would enable cognizant compilers to find it a compile time, as in the following example:

```cpp
struct my_policy: vector_execution_policy {
    static const int safelen = 8;
    static const bool vectorize_remainder = true;
};

for_loop( my_policy(), 0, 1912, [&](int i) {
    Z[i+8] = Z[i]*A;
});
```

Here, safelen is a semantic piece of information, similar to a safelen clause in OpenMP 4.0, that says that the (i+9)th application of the function cannot start until the ith and prior applications complete. For programmers to rely on this in portable code would require standardizing it.

In contrast, vectorize_remainder is a performance hint, and could remain vendor specific.

4 Wavefront Application

Our proposed vector_execution_policy gives programmers classic “vector loop” evaluation order guarantees when used with function template for_loop from P0075. We abstract the evaluation order by defining “wavefront application”. Intuitively, the wavefront application of a function f over a sequence of argument lists applies f to each argument list in a way that keeps preceding applications from falling behind later

---

3 Yes, 9 and not 8. The wavefront semantics prevent the oldest iteration in flight from getting behind the newest iteration in flight.

4The term “wavefront” for similar orderings has a long history in the field of vector and parallel programming. An example is Figure 7 from reference [4].
application. This property distinguishes our `vector_execution_policy` from our `unsequenced_execution_policy`. The wavefront property has two benefits:

- It enables exploiting “forward dependencies”, a common technique in classic vector codes.
- It implies that `vector_execution_policy` is safe to use on any loop that could be auto-vectorized.

For example, consider:

```c
void f() {
    extern float U[], V[], A, B;
    for_loop( vec, 1, 999, [&](int i) {
        V[i] = U[i+1]*A;
        U[i] = V[i-1]+B;
    });
}
```

For this code to have the same side effects with `vec` as with the `seq` policy, it is imperative that the load of `U[k]` preceded a store into `U[k]` in a later iteration, and likewise that the store into `V[k]` precede the load of `V[k]` in a later iteration. Our wavefront semantics coupled with the subscript patterns give those guarantees. With the more relaxed ordering of our `unsequenced_execution_policy` (or the existing `parallel_execution_policy` or `parallel_vector_execution_policy`) the programmer would need to fission the loop into two loops, with the consequent penalty of increasing consumption of memory bandwidth.

We optionally propose a `vec` rule to ensure that “scatters” behave in a way consistent with serial semantics. For example, given:

```c
void f() {
    extern float A[], B[];
    extern int P[], Q[];
    for_loop( vec, 0, 1000, [&](int i) {
        A[P[i]] = B[Q[i]];
    });
}
```

This “ordered scatter” rule would ensure that the result is the same as for replacing `vec` with `seq`, even if there are duplicate values in array `P`. In contrast, this example has undefined behavior if `unseq` is used and `P` has duplicate values, even if all elements of `B` are identical, because there would be unsequenced modifications of the same element of `A`. The inclusion of this rule in the standard is optional, in that the remainder of the proposal is unaffected by its presence.

Wavefront application provides the necessary conditions for vectorization on classic “long vector” machines in the tradition of Cray and Convex, vectorization on “short vector” architectures (such as Intel® SSE, Intel® AVX, ARM® NEON, and Freescale®

---

5The example is a toy, but the dependence pattern is similar to those in staggered finite-time finite-difference codes.
AltiVec), as well as software pipelining and unroll-and-interleave optimizations, without introducing relaxations that would be harmful for some loops.

4.1 Horizontal Matching

Precisely defining “ahead” and “behind” can be tricky for functions with control flow that repeats evaluation of an expression. We solve the problem by refining the sequencing rules from N4237 to handle cyclic control flow. Our refinement uses “horizontal matching” that distinguish evaluating the same expression or statement during different trips though a loop or in different invocations of a callee. Furthermore, unstructured control flows (gotos and switch statements like in “Duff’s device”) are handled by temporarily disabling synchronization guarantees across iterations, but in a way that limits the disabling to within a certain scope. While disabled, the vec policy temporarily acts like the unseq policy (i.e., the sequencing guarantees are relaxed).

Horizontal matching is fully defined and further explained in the proposed wording section (Section 7.6). For the next section, it suffices to understand that horizontal matching formalizes an intuitive notion of matching up corresponding evaluations in a sensible way. For example, given this code:

```cpp
for_loop(par, 0, 4, [&](int k){
    if (k % 2)
        f(k);
    else
        g(k);
    h(k);
}
```

the rules horizontally match each row of evaluations shown in the table below.

<table>
<thead>
<tr>
<th>Expression</th>
<th>k=0</th>
<th>k=1</th>
<th>k=2</th>
<th>k=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x % 2</td>
<td>0 % 2</td>
<td>1 % 2</td>
<td>2 % 2</td>
<td>3 % 2</td>
</tr>
<tr>
<td>f(x)</td>
<td>f(1)</td>
<td>f(2)</td>
<td>f(3)</td>
<td></td>
</tr>
<tr>
<td>g(x)</td>
<td>g(0)</td>
<td>g(1)</td>
<td>g(2)</td>
<td></td>
</tr>
<tr>
<td>h(x)</td>
<td>h(0)</td>
<td>h(1)</td>
<td>h(2)</td>
<td>h(3)</td>
</tr>
</tbody>
</table>

Executions of iterative statements are matched by matching each iteration in turn, giving up after at least one loop quits. Unstructured control-flow turns off matching until it becomes structured again. We defer the details of when this happens to the proposed wording section (7.6).

4.2 Ordering Rules for Wavefront Application

4.2.1 High-level view

The invocations of element access functions in our for_loop template from P0075 invoked with an execution policy of type vector execution policy are permitted to execute in an unordered fashion in the calling thread, unsequenced with respect to one another within the calling thread, but restricted by the “wavefront application” ordering constraints formalized in the proposed wording in Section 7.6.
Figure 1 sketches the rule for the $i^{th}$ and $j^{th}$ invocations of the element access function, where $i<j$. The subscripted letters denote expression evaluations or statement executions. Dashed lines denote “horizontally matched”; solid arrows denote “sequenced before”. Our rules require that if either black partial triangle exists, then the blue sequenced-before relationship must be enforced to complete the triangle.

![Diagram of horizontally matched and sequenced before relationships](image)

Thus the $j^{th}$ iteration cannot get ahead of the $i^{th}$ iteration.

4.2.2 Wavefront ordering for loops within the element access function

Consider the following vector for loop invocation with a serial for loop nested within the element-access function (a lambda expression, in this case):

```c
for_loop( vec, 0, 2, [&]{ int i) {
    for( int m=i; m<2; ++m )
        A[m][i] = 1;
    B[i]++;
}});
```

The definition of horizontal matching distinguishes the three evaluations of $m<2$ and two evaluations of $A[m][i]$ as five separate evaluations (in the case of $i=0$), as if the inner loop were unrolled. The dashed lines in Figure 2 show the horizontally matched relationships and the solid arrows show some of the resulting sequenced-before relationships. Evaluations of $++m$ and $1$ were omitted for brevity. Left side are evaluations for $i=0$; right side for $i=1$. As traditional with such diagrams, we omit some of the arrows inferable via transitive closure.

![Diagram of horizontal matching in a loop](image)
For two evaluations in are horizontally matched if their vertical antecedents (see proposed wording in Section 7.6) are horizontally matched. It is critical that vertical antecedent, unlike sequenced before, is not a transitive relationship, so the first evaluation of $m<2$ in the first column is not horizontally matched with the second evaluation of $m<2$ in the second column because their vertical antecedents are not horizontally matched.

If evaluations for different iterations of the inner loop were not distinguished, evaluation of the expression $m<2$ would be sequenced before $A[m][i]$ across applications and vice-versa, resulting in arrows from every expression evaluation on the left to every expression evaluation on the right, which would imply serial execution order.

Note that the rules do produce sequenced-before relationships from each evaluation within the nested loop to evaluation of $B[i]++$ immediately following the loop. This property is called “re-convergence” and is important for maximizing vector parallelism.

5 Functions for strengthening wavefront ordering

5.1 vec_off

It is sometimes useful to force serial sequencing of a region of code. We define a template function vec_off for this purpose. Here is an example:

```c
extern int* p;
for_loop(vec, 0, n, [&](int i) {
  y[i] += y[i+1];
  if(y[i]<0) {
    vec_off([]{
      *p++ = i;
    });
    *p++ = i;
  }
});
```

The updates $*p++=i$ will occur in the same order as if the policy were seq.

5.2 ordered_update

The class template ordered_update_t and function template ordered_update enable concise expression of some common patterns that require tightening the sequencing rules. Given an lvalue $x$ of type $X$, a call ordered_update($x$) returns a proxy of type ordered_update_t<$X$> that sequences assignment and update operations as if they were wrapped in vec_off. Example patterns:

```c
ordered_update(A[B[i]]) = f(i);  // Scatter
ordered_update(A[B[i]]) += f(i);  // Histogram
++ordered_update(A[B[i]]);        // Histogram
A[i] = (ordered_update(x) += f(i));  // Prefix scan
if(p(i)) A[ordered_update(j)++] = f(i);  // Compress
if(p(i)) v = A[ordered_update(j)++];  // Expand
```

P076r1 Vector and Wavefront Policies
6 Alternative Designs Considered

At the September, 2014 meeting in Urbana, the model of vector programming presented here was known as the wavefront model. Its key characteristic is that dynamically-forward loop-carried dependencies are honored without additional syntax. Two other models described in Urbana were the lock-step model and the explicit ordering-point model (also called the explicit barrier model).

N4238 provides a detailed description of these models, but they can be briefly summarized as follows:

The lock-step model groups consecutive loop iterations into chunks of known size, with execution proceeding concurrently on all iterations within a chunk as if each iteration were executing the same operation at the same time (i.e., in lock step).

The wavefront model allows iterations to proceed at different rates, but does not allow execution of one iteration to “get behind” execution of a subsequent iteration. Consequently, later iterations can depend on progress guarantees that support dynamically-forward loop-carried dependencies, as in the following example:

```c
extern float A[N];
parallel::for_loop(0, N - 1, [&](int i)
    // Evaluate f(A[i+1]) and store the result in A[i] occurs
    // before A[i+1] is modified in the next iteration.
    A[i] = f(A[i + 1]);
});
```

The explicit ordering-point model is similar to the wavefront model except that the sequencing relationships required to support dynamically-forward loop-carried dependencies would need to be made explicit by inserting ordering point constructs into the loop body, e.g., as in the following example.

```c
extern float A[N];
parallel::for_loop(0, N - 1, [&](int i)
    auto tmp = f(A[i + 1]);
    // Ensure that evaluating f(A[i+1]) occurs
    // before A[i+1] is modified in the next iteration.
    parallel::wavefront_ordering_pt();
    A[i] = f(tmp);
});
```

6.1 Previous discussions

There was consensus before Urbana that we wish our loop-like vectorization construct to have serial equivalent semantics; i.e., it should be possible to get semantically correct results by executing the code serially. This goal conflicts with the lock-step model, which requires explicit chunking of the loop and specifies a very restrictive set of valid orderings within a chunk. Moreover, lock-step execution has a semantic whereby results calculated in one iteration of the loop may be required to be available in a previous iteration of the loop. Because serial ordering is not a valid ordering with the lock-step model, the lock-step programming model was not considered appropriate as the primary vector programming paradigm in C++. Both the explicit and wavefront models do support serial ordering as a valid implementation choice.
The explicit and wavefront models both had consensus support in Urbana, with the explicit model having slightly stronger support than the wavefront model. The authors of this paper deliberated long and hard on the issue and, after considering many issues, the original authors of this proposal agreed that the wavefront model was the preferred model for vector programming, although the explicit model may still have a role to play in some sort of low-overhead parallel programming which has yet to be proposed.

In Kona (October 2015), the library syntax for vector loops proposed in P0075 was well received, in general, but the question of implicit versus explicit expression of inter-iteration dependencies remained stalled. Meetings with several hardware vendors and programmers with vectorization expertise reinforced our conclusion that the wavefront model, without explicit ordering points, best expresses vectorization as it historically understood. We did, however, learn that the “ordered scatter” rule in the first version of this paper is separable from the rest of the proposal in that some existing vector systems enforce ordered scatters whereas others do not. For this reason, we have labeled this rule as “optional” and would be willing to vote on it separately.

The remainder of this section is devoted to explaining our rationale for choosing the wavefront model over the explicit model for vector programming.

6.2 The promise and disappointments of the explicit ordering-point model

Conceptually, the explicit ordering-point model is more like a parallel programming model than is the wavefront model. An ordering point would act similar to a software barrier, preventing code motion across the ordering point but allowing it between ordering points. Theoretically, less care to maintain lexical ordering would be needed in early phases of compilation thus permitting more liberal transformations.

As we analyzed this claim of better optimization, however, we discovered some issues. To be sure, there are situations where the claim is true, but there are situations where a naïve compiler could lose optimization opportunities because the ordering points are coarse-grained, and might need to be inserted in multiple places. It is possible to make the ordering points more precise, e.g., by specifying exactly the “to” and “from” points of inter-iteration dependencies. However, this would complicate the syntax in a way that we determined was too arcane and would discourage the use of vectorization.

Moreover, some expressions that are handled naturally in the wavefront model but are difficult to express using explicit ordering points. Assuming arrays A and B and loop control variable i, the expression,

\[ A[i] = 2 \times A[i + 1]; \]

requires that A[1] in iteration 1 not be modified until its value has been read in iteration 0. With the explicit ordering-point model, an ordering point would need to be inserted between the read of A[i+1] and the modification of A[i]:

```cpp
auto tmp = A[i + 1];
parallel::wavefront_ordering_pt();
A[i] = 2*tmp;
```
Not only is the above workaround somewhat ugly and potentially error prone, but it show one of several warts that are exposed when the explicit ordering-point model is examined closely. It is not clear how many more such warts are necessary to express the entire body of vectorizable code.

Finally, the explicit model was touted as a way to express a form of parallelism more general than SIMD vectorization and software pipelining (e.g., a low-overhead parallelism that could be implemented on SIMT GPUs). While this idea has some merit, it is somewhat speculative at this point. It is not clear that the model is sufficiently rich to express the desired semantics. It is our opinion that a generalized low-overhead parallelism that can be implemented with multiple mechanisms (including SIMD) should be the subject of a future proposal, after the issues have been thoroughly explored, and with a couple of implementations. We should not hold up support for vectorization pending such exploration.

6.3 Existing Practice

The wavefront model is a formalization of the model that has been used for SIMD and long-vector architectures for decades [1][2][2]. It has been analyzed and refined in the technical literature and it has been implemented in many compilers and in many programming languages including C, C++, and Fortran (via OpenMP as well as proprietary annotations).

The experts in vector programming are familiar with the wavefront model; to them, it’s what vector programming looks like. Even if we were to all agree that the explicit model is easier to learn than the wavefront model (and that is certainly not obvious), we don’t want to standardize something that is hostile to experts.

6.4 Using vec with Other Algorithms

We considered applying vec to all algorithms in the Parallelism TS but we felt that it was not clear what that would mean and that assigning an arbitrary meaning would give the programmer a mistaken impression of usability. We might give vec a meaning to more algorithms in the future, if and when we identify a reasonable meaning for them.

7 C++ Proposed Wording

The proposed edits are with respect to the current Parallelism TS assuming the adoption of P0075.

7.1 Feature test macros

Add the following row to Table 1 in section 1.5 [parallel.general.features]

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Header</th>
</tr>
</thead>
<tbody>
<tr>
<td>__cpp_lib_experimental_vector_execution_policy</td>
<td>201602</td>
<td>&lt;experimental/algorithm&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt;experimental/execution_policy&gt;</td>
</tr>
</tbody>
</table>

P0076r1 Vector and Wavefront Policies
Editorial note: The format of this section of the TS should probably be changed to match that of the Library Fundamentals TS, which has a 6-column table that includes the name of the specific feature and the document number that proposed it.

7.2 Header <experimental/execution_policy> synopsis

Add the following to section [parallel.execpol.synopsis]:

```cpp
class vector_execution_policy;
class unsequenced_execution_policy;
```

7.3 Add new execution policies

Rename section 2.6:

**2.6 Parallel+Vector unsequenced execution policy [parallel.execpol.parvec]**

And add the following subsections:

**2.x Vector execution policy [parallel.execpol.vec]**

```cpp
class vector_execution_policy{ unspecified };
```

The class `vector_execution_policy` is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and indicate that a parallel algorithm's execution may be vectorized, but must respect wavefront evaluation order.

**2.x Unsequenced execution policy [parallel.execpol.unseq]**

```cpp
class unsequenced_execution_policy{ unspecified };
```

The class `unsequenced_execution_policy` is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and indicate that a parallel algorithm's execution may be vectorized.

7.4 Execution policy objects

Add to [parallel.execpol.objects]:

```cpp
constexpr vector_execution_policy vec{};
constexpr unsequenced_execution_policy unseq{};
```

7.5 Exception reporting behavior

Edit 3.1 [parallel.exceptions.behavior] paragraph 2 as shown:

- If the execution policy object is of type class `vector_execution_policy`, `unsequenced_execution_policy`, or `parallel_vector_execution_policy`, `std::terminate` shall be called.

7.6 Wavefront Application

New subsection to add to section 4.1. Shaded text is explanatory and not part of the formal wording.
Wavefront Application [parallel.alg.general.wavefront]

For the purpose of this section, an evaluation is a value computation or side effect of an expression or execution of a statement. Initialization of a temporary object is considered a subexpression of the expression that necessitates the temporary object.

An evaluation A contains an evaluation B if evaluation of B occurs as part of evaluation of A. [Note: This includes evaluations occurring in function invocations. -- end note]

Vertical antecedent is an irreflexive, nonsymmetric, nontransitive relationship between two evaluations. For an evaluation A sequenced before an evaluation B, both contained in the same invocation of an element access function, A is a vertical antecedent of B if:

- there exists an evaluation S such that:
  - S contains A, and
  - S contains all evaluations C (if any) such that A is sequenced before C and C is sequenced before B,
  - but S does not contain B, and
- control reached B from A without executing any of the following:
  - a goto statement that jumps to a statement outside of S, or
  - a switch statement executed within S that transfers control into a substatement of a nested selection or iteration statement, or
  - a throw [Note: even if caught – end note], or
  - a longjmp.

The first major bullet above describes what could informally be described as “immediately precedes”. If A and B are part of the same statement, then A is a vertical antecedent of B only if there is nothing sequenced between them. If A and B are part of different statements, then A is a vertical antecedent of B if, by popping out zero or more levels of nesting, you find a point where the statement containing A immediately precedes B. This is the point of re-convergence after a control-flow divergence.

The second major bullet is needed to handle cases where re-convergence is difficult or impossible to establish. In those cases, the guarantees degenerate to those provided by the unsequenced_execution_policy until convergence is re-established at the end of the block containing both the jump statement and the jumped-to statement.

In the following, $X_i$ and $X_j$ refer to evaluations of the same expression or statement contained in the application of an element access function corresponding to the $i^{th}$ and $j^{th}$ elements of the input sequence. [Note: There might be several evaluations $X_k$, $Y_k$, etc. of a single expression or statement in application $k$, for example, if the expression or statement appears in a loop within the element access function. -- end note]
**Horizontally matched** is an equivalence relationship between two evaluations of the same expression. An evaluation $B_i$ is *horizontally matched* with an evaluation $B_j$ if:

- neither evaluation has a vertical antecedent, or
- there exist horizontally matched evaluations $A_i$ and $A_j$ that are vertical antecedents of evaluations $B_i$ and $B_j$, respectively.

The rules for establishing the horizontally matched relationship match evaluations in one application with corresponding evaluations in a separate application of the element access function. The nature of the rules are such that even nested loops work correctly. For example, given:

```plaintext
b;
while ( e )
  stmt;
c;
```

where $b_i$ is a horizontal antecedent of $b_j$. Intuitively, we would expect the kth evaluation of $e_i$ to be the horizontal antecedent of the kth evaluation of $e_j$, assuming both evaluations happen. Even if one of the invocations executes $e$ more times than the other, all evaluations of $e_i$ and $e_j$ are vertical antecedents of $c_i$ and $c_j$, respectively, so the horizontal antecedent relationship is re-established for $c_i$ and $c_j$.

Let $f$ be a function called for each argument list in a sequence of argument lists. *Wavefront application of $f$* requires that evaluation $A_i$ be sequenced before evaluation $B_j$ if:

- $A_i$ is sequenced before some evaluation $B_i$ and $B_i$ is horizontally matched with $B_j$, or
- $A_i$ is horizontally matched with some evaluation $A_j$ and $A_j$ is sequenced before $B_j$.

[Note: The relationships between $A_i$ and $B_i$ and between $A_j$ and $B_j$ are *sequenced before*, not *vertical antecedent*. -- end note]

The two bullets describe the two triangles in Figure 1.

### 7.7 Optional clause for ordered scatters

The following rule requires ordering of side effects in a way that supports overlapping scatters without use of the `ordered_update()` function. It is useful, but not essential, for vector programming and could be removed from this paper without damaging the rest of the proposal.

Continuing the previous section, add:

The *direct side effects* of an expression $X$ are those caused by evaluating $X$, but not including side effects caused by evaluating its sub-expressions. For any two evaluations $A_i$ and $A_j$ such that $A_i$ is a horizontal antecedent of $A_j$, all direct side effects in $A_i$ are sequenced before all direct side effects in $A_j$. 

P0076r1 Vector and Wavefront Policies
This clause allows for code such as:

\[
U[V[i]] = expr(i);
\]

to produce deterministic results even if \( V[i] \) contains duplicate elements (sometimes called the overlapping scatter pattern).

If this clause is adopted, we will also want a library function, unordered_update, having a syntax similar to ordered_update, that relaxes this guarantee and allows the generation of faster code on architectures with scatter instructions that do not support ordered writes. The unordered_update function should be used only when \( V[i] \) is known not to contain duplicates.

7.8 Effect of execution policies on algorithm execution

To section 4.1.2 [parallel.alg.general.exec], add:

The invocations of element access functions in parallel algorithms invoked with an execution policy of type unsequenced_execution_policy are permitted to execute in an unordered fashion in the calling thread, unsequenced with respect to one another within the calling thread.

The invocations of element access functions in parallel algorithms invoked with an execution policy of type vector_execution_policy are permitted to execute in an unordered fashion in the calling thread, unsequenced with respect to one another within the calling thread, subject to the constraints of wavefront application order for the last argument to for_loop or for_loop_strided.

7.9 Header <experimental/algorithm> synopsis

Add the following to 4.3.1 [parallel.alg.ops.synopsis]:

```cpp
namespace std {
 namespace experimental {
 namespace parallel {
 inline namespace v2 {

 template<typename F>
 auto vec_off(F&& f) -> decltype(f());

 template<class T>
 class ordered_update_t;

 template <class T>
 ordered_update_t<T> ordered_update(T& ref);

 }}}}

7.10 vec_off

Add this function to section 4.3 [parallel.alg.ops]:

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4.3.x Vec off [parallel.alg.vecoff]

```cpp
template<typename F>
auto vec_off(F&& f) -> decltype(f());
```

**Effects:** Evaluates `std::forward<F>(f)()`. If two calls to `vec_off` are horizontally matched within a wavefront application of an element access function over input sequence S, then the evaluation of `f()` in the application for one element in S is sequenced before the evaluation `f()` in the application for a subsequent element in S; otherwise (for other execution policies) there is no effect on sequencing.

**Returns:** the result of the evaluation of `f()`.

7.11 ordered_update

Add these subsections to section 4.2 [parallel.alg.ops]

4.3.x Ordered update class [parallel.alg.ordupdate.class]

```cpp
template<class T>
class ordered_update_t {
    T& ref;  // exposition only
public:
    ordered_update_t(T& loc);
    template <class U>
    auto operator=(U rhs);
    template <class U>
    auto operator+=(U rhs);
    template <class U>
    auto operator-=(U rhs);
    template <class U>
    auto operator*=(U rhs);
    template <class U>
    auto operator/=(U rhs);
    template <class U>
    auto operator%=(U rhs);
    template <class U>
    auto operator>>=(U rhs);
    template <class U>
    auto operator<<=(U rhs);
    template <class U>
    auto operator&(U rhs);
    template <class U>
    auto operator^(U rhs);
    template <class U>
    auto operator|(U rhs);
    auto operator++();
    auto operator++(int);
    auto operator--();
    auto operator--(int);
};
```

An object of type `ordered_update_t<T>` is a proxy for an object of type `T` intended to be used within a parallel application of an element access function using a policy object of type `vector_execution_policy`. Simple assignments and compound assignments to the object are forwarded to proxied object, but are sequenced as though executed within a `vec_off` invocation.
ordered_update_t(T& loc);

   Effect: Initialize ref with loc.

template <class U>
   auto operator=(U rhs);

   Returns: equivalent to vec_off([&]{ return ref = std::move(rhs); })

template <class U>
   auto operator+=(U rhs)-> decltype(ref+=rhs);

template <class U>
   auto operator-=(U rhs);

template <class U>
   auto operator*=(U rhs);

template <class U>
   auto operator/=(U rhs);

template <class U>
   auto operator%=(U rhs);

template <class U>
   auto operator>>=(U rhs);

template <class U>
   auto operator<<=(U rhs);

template <class U>
   auto operator&=(U rhs);

template <class U>
   auto operator^=(U rhs);

template <class U>
   auto operator|=(U rhs);

   Returns: for the respective binary operator op (one of +, -, *, /, %, >>, <<, &, ^, or |),
            equivalent to vec_off([&]{ return ref op= std::move(rhs); })

auto operator++();
   Returns: equivalent to vec_off([&]{ return ++ref; })

auto operator++(int);
   Returns: equivalent to vec_off([&]{ return ref++; })

auto operator--();
   Returns: equivalent to vec_off([&]{ return --ref; })

auto operator--(int);
   Returns: equivalent to vec_off([&]{ return ref--; })

4.3.x Ordered update function template [parallel.alg.ordupdate.func]

   template <class T>
      ordered_update_t<T> ordered_update(T& ref);

   Returns: ordered_update_t<T>(ref)

Optional: If the implicit scatter rule is included, then we will want a way to turn it off
when it is not required. If the committee goes in that direction, we will add a function
unordered_update that turns the rule off, similar in style to how ordered_update turns
the rule on.
8 Acknowledgement

Olivier Giroux provided the ideas behind “horizontally matched” and “vertical antecedent”.

9 References


[2] Lee Higbie, Vectorization and Conversion of Fortran Programs for the CRAY-1 (CFG) Compiler, Undated, but seems to be from Cray-1 timeframe. PDF page 15 describes vectorization of a loop with a forward lexical dependence.

[3] Cray Assembly Language (CAL) for Cray X1 Systems Reference Manual, Section 2.6 says “Otherwise, the Cray X1 system guarantees that B will reference memory after A only if: ... A and B are elements of the same ordered vector scatter or zero-stride vector store.”

