Hash Tables for the
Standard Template Library

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1 Introduction

The Standard Template Library (STL), which has been approved for inclusion in the C++ Standard Library by the ANSI/ISO C++ Standards Committee, supplies a set of associative container classes (set, multiset, map, multimap) that support fast storage retrieval of data based on keys. As specified in the STL standard these containers could be more precisely described as sorted associative containers, since in addition to fast storage and retrieval they are required to support efficient iteration through the entries in sorted order—according to the order determined by a boolean-valued function supplied as a parameter when the container is created. The sorted iteration capability is not always needed. If the requirement for fast sorted iteration is dropped, a wider range of implementations is possible; in particular, an attractive choice is a hash table implementation. In this report we first show how to restructure the existing STL associative container requirements into two parts: (1) base requirements for all associative containers, and (2) an additional set of requirements for sorting capabilities. We then introduce a set of requirements (3) for a hashing capability that can be added to (1) to produce a set of requirements for hash tables. These additions are fully consistent with the STL framework and also serve as a significant example of the extensibility of the framework.

We have developed and experimented with two independent hash table implementations, which will be described in detail in a separate report. These implementations not only demonstrate the maturity and consistency of the requirements described here, but should also be of immediate practical use to C++ programmers who need associative containers but can do without sorted order of the keys. These implementations will be available on or before February 20, 1995.

Part of the material in the following section is adapted directly from The Standard Template Library, by Alexander Stepanov and Meng Lee, Hewlett-Packard Report, December 6, 1994. This section is the restructuring and extension of the section on associative containers (Section 8) from that document. The intended changes are the following:

- The name “Associative containers” has been changed to “Sorted associative containers”
- The requirements for Associative containers have been divided into a base set which is shared with unordered associations such as hashing
- A new requirements table has been added to define the requirements for hash tables.

The interfaces of container classes set, multiset, map, and multimap have been included for completeness; no changes are being proposed to these classes.

2 Associative containers

Associative containers provide an ability for fast retrieval of data based on keys. There are two major categories of associative containers: sorted associative containers and hashed associative containers; the latter are also called hash tables. These categories share many requirements, which we call the base requirements for associative containers.

The library provides four basic kinds of sorted associative containers: set, multiset, map and multimap, and four basic kinds of hash tables: hash_set, hash_multiset, hash_map and hash_multimap.

All of the sorted associative containers are parameterized on Key and an ordering relation Compare that induces a total ordering on elements of Key. In addition, map and multimap associate an arbitrary type T with the Key. The object of type Compare is called the comparison object of a container.
All of the hash tables are parameterized on `Key`, a function type `Hasher` that maps elements of `Key` into integers, and a function type `KeyEqual` that induces an equivalence relation on elements of `Key`. In addition, `hash_map` and `hash_multimap` associate an arbitrary type `T` with the `Key`. The object of type `KeyEqual` is called the *key-equivalence object* of a container.

With sorted associative containers, when we talk about equality of keys we mean the equivalence relation imposed by the comparison and not the `operator==` on keys. That is, two keys `k1` and `k2` are considered to be equal if for the comparison object `comp, comp(k1, k2) == false && comp(k2, k1) == false`.

With hash tables, when we talk about equality of keys we mean the equivalence relation imposed by the key-equivalence object and not the `operator==` on keys. That is, two keys `k1` and `k2` are considered to be equal if for the key equivalence object `equal, equal(k1, k2) == true`.

An associative container (either sorted or hashed) supports *unique keys* if it may contain at most one element for each key. Otherwise, it supports *equal keys*. `set, map, hash_set, and hash_map` support unique keys. `multiset, multimap, hash_multiset, and hash_multimap` support equal keys.

For `set, multiset, hash_set, and hash_multiset`, the value type is the same as the key type. For `map, multimap, hash_map, and hash_multimap`, it is equal to `pair<const Key, T>`.

The iterator type of a sorted associative container is of the bidirectional iterator category, while that of hash tables is of the forward iterator category. `insert` does not affect the validity of iterators and references to the container, and `erase` invalidates only the iterators and references to the erased elements.

In the following table, `X` is an associative container class, `a` is a value of `X, a_uniq` is a value of `X` when `X` supports unique keys, and `a_eq` is a value of `X` when `X` supports multiple keys, `i` and `j` satisfy input iterator requirements and refer to elements of `value_type`, `[i, j)` is a valid range, `p` is a valid iterator to `a, q, q1, q2` are valid dereferenceable iterators to `a, [q1, q2)` is a valid range, `t` is a value of `X::value_type and k` is a value of `X::key_type`.

**Table 1: Base requirements for associative container (in addition to container)**

<table>
<thead>
<tr>
<th>expression</th>
<th>return type</th>
<th>assertion/note</th>
<th>complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>X::key_type</code></td>
<td>Key</td>
<td></td>
<td>compile time</td>
</tr>
<tr>
<td><code>a_uniq.insert(t)</code></td>
<td><code>pair&lt;iterator, bool&gt;</code></td>
<td>inserts <code>t</code> if and only if there is no element in the container with key equal to the key of <code>t</code>. The <code>bool</code> component of the returned pair indicates whether the insertion takes place and the <code>iterator</code> component of the pair points to the element with key equal to the key of <code>t</code>.</td>
<td>logarithmic</td>
</tr>
<tr>
<td><code>a_eq.insert(t)</code></td>
<td>iterator</td>
<td>inserts <code>t</code> and returns the iterator pointing to the newly inserted element.</td>
<td>logarithmic</td>
</tr>
<tr>
<td><code>a.insert(i, j)</code></td>
<td>result is not used</td>
<td>inserts the elements from the range <code>[i, j)</code> into the container.</td>
<td>(N\log(\text{size}()+N)) ((N) is the distance from <code>i</code> to <code>j</code>) in general; linear if <code>[i, j)</code> is sorted according to <code>value_comp()</code></td>
</tr>
</tbody>
</table>
2.1 Sorted Associative Containers

The additional requirements for sorted associative containers are given in the following table:

Table 2: Sorted associative container requirements (in addition to base requirements)

<table>
<thead>
<tr>
<th>expression</th>
<th>return type</th>
<th>assertion/note</th>
<th>complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.erase(k)</td>
<td>size_type</td>
<td>erases all the elements in the container with key equal to ( k ). returns the number of erased elements.</td>
<td>( \log(\text{size}()) + \text{count}(k) )</td>
</tr>
<tr>
<td>a.erase(q)</td>
<td>result is not used</td>
<td>erases the element pointed to by ( q ).</td>
<td>amortized constant</td>
</tr>
<tr>
<td>a.erase(q1, q2)</td>
<td>result is not used</td>
<td>erases all the elements in the range ([q1, q2)).</td>
<td>( \log(\text{size}()) + N ) where ( N ) is the distance from ( q1 ) to ( q2 ).</td>
</tr>
<tr>
<td>a.find(k)</td>
<td>iterator; const_iterator for constant ( a )</td>
<td>returns an iterator pointing to an element with the key equal to ( k ), or ( a.\text{end()} ) if such an element is not found.</td>
<td>logarithmic</td>
</tr>
<tr>
<td>a.count(k)</td>
<td>size_type</td>
<td>returns the number of elements with key equal to ( k ).</td>
<td>( \log(\text{size}()) + \text{count}(k) )</td>
</tr>
<tr>
<td>a.equal_range(k)</td>
<td>pair&lt;iterator, iterator&gt;; pair&lt;const_iterator, const_iterator&gt; for constant ( a )</td>
<td>returns a pair where the first iterator points to the first element having key equal to ( k ), and the second element being the first element beyond all having key ( k ). The range is empty if no elements have key ( k ).</td>
<td>logarithmic</td>
</tr>
</tbody>
</table>
The fundamental property of iterators of sorted associative containers is that they iterate through the containers in the non-descending order of keys, where non-descending is defined by the comparison that was used to construct them. For any two dereferenceable iterators $i$ and $j$ such that distance from $i$ to $j$ is positive,

$$\text{value\_comp}(\ast j, \ast i) == \text{false}$$

For sorted associative containers with unique keys the stronger condition holds,

$$\text{value\_comp}(\ast i, \ast j) == \text{true.}$$

### 2.1.1 Set

set is a kind of sorted associative container that supports unique keys (contains at most one of each key value) and provides for fast retrieval of the keys themselves.

<table>
<thead>
<tr>
<th>expression</th>
<th>return type</th>
<th>assertion/note</th>
<th>complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(i, j, c)$ $X a(i, j, c)$;</td>
<td>constructs an empty container and inserts elements from the range $[i, j)$ into it; uses $c$ as a comparison object.</td>
<td></td>
<td>NlogN in general (N is the distance from $i$ to $j$); linear if $[i, j)$ is sorted with value_comp()</td>
</tr>
<tr>
<td>$X(i, j)$ $X a(i, j)$;</td>
<td>same as above, but uses Compare() as a comparison object.</td>
<td></td>
<td>same as above</td>
</tr>
<tr>
<td>a.key_comp()</td>
<td>$X::\text{key_compare}$</td>
<td>returns the comparison object out of which $a$ was constructed.</td>
<td>constant</td>
</tr>
<tr>
<td>a.value_comp()</td>
<td>$X::\text{value_compare}$</td>
<td>returns an object of value_compare constructed out of the comparison object.</td>
<td>constant</td>
</tr>
<tr>
<td>a.insert(p, t)</td>
<td>iterator</td>
<td>inserts $t$ if and only if there is no element with key equal to the key of $t$ in containers with unique keys; always inserts $t$ in containers with equal keys. always returns the iterator pointing to the element with key equal to the key of $t$. iterator $p$ is a hint pointing to where the insert should start to search.</td>
<td>logarithmic in general, but amortized constant if $t$ is inserted right after $p$.</td>
</tr>
<tr>
<td>a.lower_bound(k)</td>
<td>iterator; const_iterator for constant $a$</td>
<td>returns an iterator pointing to the first element with key not less than $k$.</td>
<td>logarithmic</td>
</tr>
<tr>
<td>a.upper_bound(k)</td>
<td>iterator; const_iterator for constant $a$</td>
<td>returns an iterator pointing to the first element with key greater than $k$.</td>
<td>logarithmic</td>
</tr>
<tr>
<td>a.equal_range(k)</td>
<td>same as base requirements</td>
<td>equivalent to make_pair(a.lower_bound(k), a.upper_bound(k)).</td>
<td>logarithmic</td>
</tr>
</tbody>
</table>
template <class Key, class Compare = less<Key>,
        template <class U> class Allocator = allocator>
class set {
public:

  // typedefs:
  typedef Key key_type;
  typedef Key value_type;
  typedef Allocator<Key>::pointer pointer;
  typedef Allocator<Key>::reference reference;
  typedef Allocator<Key>::const_reference const_reference;
  typedef Compare key_compare;
  typedef Compare value_compare;
  typedef iterator;
  typedef iterator const_iterator;
  typedef size_type;
  typedef difference_type;
  typedef reverse_iterator;
  typedef const_reverse_iterator;

  // allocation/deallocation:
  set(const Compare& comp = Compare());
  template <class InputIterator>
  set(InputIterator first, InputIterator last,
  const Compare& comp = Compare());
  set(const set<Key, Compare, Allocator>& x);
  ~set();
  set<Key, Compare, Allocator>& operator=(const set<Key, Compare,
  Allocator>& x);
  void swap(set<Key, Compare, Allocator>& x);

  // accessors:
  key_compare key_comp() const;
  value_compare value_comp() const;
  iterator begin() const;
  iterator end() const;
  reverse_iterator rbegin();
  reverse_iterator rend();
  bool empty() const;
  size_type size() const;
  size_type max_size() const;

  // insert/erase:
  pair<iterator, bool> insert(const value_type& x);
  iterator insert(iterator position, const value_type& x);
  template <class InputIterator>
  void insert(InputIterator first, InputIterator last);
  void erase(iterator position);
  size_type erase(const key_type& x);
  void erase(iterator first, iterator last);

  // set operations:
  iterator find(const key_type& x) const;
  size_type count(const key_type& x) const;
iterator lower_bound(const key_type& x) const;
iterator upper_bound(const key_type& x) const;
pair<iterator, iterator> equal_range(const key_type& x) const;
};

template <class Key, class Compare, class Allocator>
bool operator==(const set<Key, Compare, Allocator>& x,
        const set<Key, Compare, Allocator>& y);

template <class Key, class Compare, class Allocator>
bool operator<(const set<Key, Compare, Allocator>& x,
        const set<Key, Compare, Allocator>& y);

iterator is a constant bidirectional iterator referring to const value_type. The exact type is implementation dependent and determined by Allocator.

const_iterator is the same type as iterator.

size_type is an unsigned integral type. The exact type is implementation dependent and determined by Allocator.

difference_type is a signed integral type. The exact type is implementation dependent and determined by Allocator.

### 2.1.2 Multiset

multiset is a kind of sorted associative container that supports equal keys (possibly contains multiple copies of the same key value) and provides for fast retrieval of the keys themselves.

template <class Key, class Compare = less<Key>,
        template <class U> class Allocator = allocator>
class multiset {
public:

    // typedefs:
    
    typedef Key key_type;
    typedef Key value_type;
    typedef Allocator<Key>::pointer pointer;
    typedef Allocator<Key>::reference reference;
    typedef Allocator<Key>::const_reference const_reference;
    typedef Compare key_compare;
    typedef Compare value_compare;
    typedef iterator;
    typedef iterator const_iterator;
    typedef size_type;
    typedef difference_type;
    typedef reverse_iterator;
    typedef const_reverse_iterator;

    // allocation/deallocation:
    multiset(const Compare& comp = Compare());
    template <class InputIterator>
    multiset(InputIterator first, InputIterator last,
                const Compare& comp = Compare());
    multiset(const multiset<Key, Compare, Allocator>& x);
    ~multiset();
    multiset<Key, Compare, Allocator>& operator=(const multiset<Key, Compare, Allocator>& x);
void swap(multiset<Key, Compare, Allocator>& x);

// accessors:

key_compare key_comp() const;
value_compare value_comp() const;
iterator begin() const;
iterator end() const;
reverse_iterator rbegin();
reverse_iterator rend();
bool empty() const;
size_type size() const;
size_type max_size() const;

// insert/erase:

iterator insert(const value_type& x);
iterator insert(iterator position, const value_type& x);
template <class InputIterator>
void insert(InputIterator first, InputIterator last);
void erase(iterator position);
size_type erase(const key_type& x);
void erase(iterator first, iterator last);

// multiset operations:

iterator find(const key_type& x) const;
size_type count(const key_type& x) const;
iterator lower_bound(const key_type& x) const;
iterator upper_bound(const key_type& x) const;
pair<iterator, iterator> equal_range(const key_type& x) const;

};

template <class Key, class Compare, class Allocator>
bool operator==(const multiset<Key, Compare, Allocator>& x, const multiset<Key, Compare, Allocator>& y);

template <class Key, class Compare, class Allocator>
bool operator<(const multiset<Key, Compare, Allocator>& x, const multiset<Key, Compare, Allocator>& y);

iterator is a constant bidirectional iterator referring to const value_type. The exact type is implementation dependent and determined by Allocator.

cost_iterator is the same type as iterator.

size_type is an unsigned integral type. The exact type is implementation dependent and determined by Allocator.

difference_type is a signed integral type. The exact type is implementation dependent and determined by Allocator.

2.1.3 Map

map is a kind of sorted associative container that supports unique keys (contains at most one of each key value) and provides for fast retrieval of values of another type T based on the keys.
public:

// typedefs:

typedef Key key_type;
typedef pair<const Key, T> value_type;
typedef Compare key_compare;
class value_compare
  : public binary_function<value_type, value_type, bool> { 
    friend class map;
  protected:
    Compare comp;
    value_compare(Compare c) : comp(c) {} 
  public:
    bool operator()(const value_type& x, const value_type& y) {
      return comp(x.first, y.first);
    }
  }; 
typedef iterator;
typedef const_iterator;
typedef Allocator<value_type>::pointer pointer;
typedef Allocator<value_type>::reference reference;
typedef Allocator<value_type>::const_reference const_reference;
typedef size_type;
typedef difference_type;
typedef reverse_iterator;
typedef const_reverse_iterator;

// allocation/deallocation:

map(const Compare& comp = Compare());

map(InputIterator first, InputIterator last,
    const Compare& comp = Compare());

map(const map<Key, T, Compare, Allocator>& x);

~map();

map<Key, T, Compare, Allocator>&
operator=(const map<Key, T, Compare, Allocator>& x);

void swap(map<Key, T, Compare, Allocator>& x);

// accessors:

key_compare key_comp() const;

value_compare value_comp() const;

iterator begin();

const_iterator begin() const;

iterator end();

const_iterator end() const;

reverse_iterator rbegin();

reverse_iterator rend();

bool empty() const;

size_type size() const;

size_type max_size() const;

Allocator<T>::reference operator[](const key_type& x); 

// insert/erase:

pair<iterator, bool> insert(const value_type& x);

iterator insert(iterator position, const value_type& x);
template <class InputIterator>
void insert(InputIterator first, InputIterator last);
void erase(iterator position);
size_type erase(const key_type& x);
void erase(iterator first, iterator last);

// map operations:
iterator find(const key_type& x);
const_iterator find(const key_type& x) const;
size_type count(const key_type& x) const;
iterator lower_bound(const key_type& x);
const_iterator lower_bound(const key_type& x) const;
iterator upper_bound(const key_type& x);
const_iterator upper_bound(const key_type& x) const;
pair<iterator, iterator> equal_range(const key_type& x);
pair<const_iterator, const_iterator> equal_range(const key_type& x) const;

};

template <class Key, class T, class Compare, class Allocator>
bool operator==(const map<Key, T, Compare, Allocator>& x,
const map<Key, T, Compare, Allocator>& y);

template <class Key, class T, class Compare, class Allocator>
bool operator<(const map<Key, T, Compare, Allocator>& x,
const map<Key, T, Compare, Allocator>& y);

iterator is a bidirectional iterator referring to value_type. The exact type is implementation dependent
and determined by Allocator.

const_iterator is a constant bidirectional iterator referring to const value_type. The exact type is
implementation dependent and determined by Allocator. It is guaranteed that there is a constructor for
const_iterator out of iterator.

size_type is an unsigned integral type. The exact type is implementation dependent and determined by
Allocator.

difference_type is a signed integral type. The exact type is implementation dependent and determined by
Allocator.

In addition to the standard set of member functions of sorted associative containers, map provides
reference operator[](const key_type&). For a map m and key k, m[k] is semantically equivalent to
(*((m.insert(make_pair(k, T()))).first)).second.

2.1.4 Multimap

multimap is a kind of sorted associative container that supports equal keys (possibly contains multiple
copies of the same key value) and provides for fast retrieval of values of another type T based on the keys.

template <class Key, class T, class Compare = less<Key>,
template <class U> class Allocator = allocator>
class multimap {
public:

// typedefs:

typedef Key key_type;
typedef pair<const Key, T> value_type;
typedef Compare key_compare;
class value_compare
public binary_function<value_type, value_type, bool> { 
friend class multimap;
protected:
    Compare comp;
    value_compare(Compare c) : comp(c) {}
public:
    bool operator()(const value_type& x, const value_type& y) {
        return comp(x.first, y.first);
    }
};
typedef iterator;
typedef const_iterator;
typedef Allocator<value_type>::pointer pointer;
typedef Allocator<value_type>::reference reference;
typedef Allocator<value_type>::const_reference const_reference;
typedef size_type;
typedef difference_type;
typedef reverse_iterator;
typedef const_reverse_iterator;

// allocation/deallocation:
multimap(const Compare& comp = Compare());
template <class InputIterator>
multimap(InputIterator first, InputIterator last,
    const Compare& comp = Compare());
multimap(const multimap<Key, T, Compare, Allocator>& x);
~multimap();
multimap<Key, T, Compare, Allocator>&
    operator=(const multimap<Key, T, Compare, Allocator>& x);
void swap(multimap<Key, T, Compare, Allocator>& x);

// accessors:
key_compare key_comp() const;
value_compare value_comp() const;
iterator begin();
const_iterator begin() const;
iterator end();
const_iterator end() const;
reverse_iterator rbegin();
reverse_iterator rend();
bool empty() const;
size_type size() const;
size_type max_size() const;

// insert/erase:
iterator insert(const value_type& x);
iterator insert(iterator position, const value_type& x);
template <class InputIterator>
void insert(InputIterator first, InputIterator last);
void erase(iterator position);
size_type erase(const key_type& x);
void erase(iterator first, iterator last);

// multimap operations:
iterator find(const key_type& x);
const_iterator find(const key_type& x) const;
size_type count(const key_type& x) const;
iterator lower_bound(const key_type& x);
const_iterator lower_bound(const key_type& x) const;
iterator upper_bound(const key_type& x);
const_iterator upper_bound(const key_type& x) const;
pair<iterator, iterator> equal_range(const key_type& x);
pair<const_iterator, const_iterator> equal_range(const key_type& x) const;

};

template <class Key, class T, class Compare, class Allocator>
bool operator==(const multimap<Key, T, Compare, Allocator>& x,
        const multimap<Key, T, Compare, Allocator>& y);

template <class Key, class T, class Compare, class Allocator>
bool operator<(const multimap<Key, T, Compare, Allocator>& x,
        const multimap<Key, T, Compare, Allocator>& y);

iterator is a bidirectional iterator referring to value_type. The exact type is implementation dependent
and determined by Allocator.

cost_iterator is the a constant bidirectional iterator referring to const value_type. The exact type is
implementation dependent and determined by Allocator. It is guaranteed that there is a constructor for
const_iterator out of iterator.

size_type is an unsigned integral type. The exact type is implementation dependent and determined by
Allocator.

difference_type is a signed integral type. The exact type is implementation dependent and determined by
Allocator.

2.2 Hash Tables

The following table gives the requirements beyond the base requirements of Table 1 for hashed associative
containers (hash tables). In this table b is an integer of type size_type, and h is a hash function of type
X::hash_type.

<table>
<thead>
<tr>
<th>expression</th>
<th>return type</th>
<th>assertion/note pre/post-condition</th>
<th>complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>X::key_equal</td>
<td>KeyEqual</td>
<td>Type of the key equality function. Defaults to equal_to&lt;key_type&gt;.</td>
<td>compile time</td>
</tr>
<tr>
<td>X::hasher</td>
<td>Hasher</td>
<td>Type of the hash function. Defaults to hash&lt;Key&gt;</td>
<td>compile time</td>
</tr>
<tr>
<td>X()</td>
<td>X a;</td>
<td>constructs an empty container; uses HashFunction() as the hash function and Compare() as the key comparison builds at least 1000 buckets.</td>
<td>constant</td>
</tr>
<tr>
<td>X(b)</td>
<td>X a(b);</td>
<td>constructs an empty container using above defaults, except having space for b buckets.</td>
<td>constant</td>
</tr>
<tr>
<td>X(b, h)</td>
<td>X a(b, h);</td>
<td>as above, but uses h as the hash function.</td>
<td>constant</td>
</tr>
</tbody>
</table>
For hash tables, iteration through the table presents the keys in no particular order; the actual order depends on the order of insertion and on the hash function. Table entries which have equal keys will appear consecutively when iterating through the table. Iterators and references remain valid after all operations, including `resize`, but ranges such as the result of `equal_range` may no longer be valid.

The bucket parameters to the constructors and the `resize` operation guide the implementation in selecting the actual number of buckets being used. An implementation is free to choose more. The number of buckets in use for a table may change any time elements are inserted or deleted from the table. The `resize(buckets)`
member function can be issued to change the bucket count at a time when the program can accommodate the overhead of table restructuring.

The hash function is expected to accept a single parameter, the key, and return a number in the range defined by size_type. The ability of the implementation to achieve constant performance depends on the ability of the hashing function to map key values across the unsigned integers.

2.2.1 Hash Function

Each implementation will provide a default hash function, a subclass of unary_function<T, size_type>, where T is the key type.

```cpp
template <class T> class hash: unary_function<T, size_type>{
    size_type operator() (const T & key);
};
```

This shall be defined at least for the numeric types and string.

2.2.2 Hash_set

hash_set is a kind of hash table that supports unique keys (contains at most one of each key value) and provides for fast retrieval of the keys themselves.

```cpp
template <class Key, class Hasher=hash<Key>, class KeyEqual = equal_to<Key>,
          template <class U> class Allocator = allocator>
class hash_set {
    public:
        // typedefs:
        typedef Key key_type;
        typedef Key value_type;
        typedef Hasher hasher;
        typedef KeyEqual key_equal;
        typedef pointer;
        typedef reference;
        typedef const_reference;
        typedef iterator;
        typedef iterator const_iterator;
        typedef size_type;
        typedef difference_type;

        // allocation/deallocation
        hash_set(size_type table_size = 1007,
                 const Hasher& hf = Hasher(),
                 const KeyEqual& equal = KeyEqual());
        template <class InputIterator>
        hash_set(InputIterator first, InputIterator last,
                 size_type table_size = 10007,
                 const Hasher& hf = Hasher(),
                 const KeyEqual& equal = KeyEqual());
        ~hash_set();

        // copying, assignment, swap
        hash_set(const hash_set<Key, Hasher, KeyEqual, Allocator>& x);
        hash_set<Key, Hasher, KeyEqual, Allocator>&
```
operator=(const hash_set<Key, Hasher, KeyEqual, Allocator>& x);
void swap(hash_set<Key, Hasher, KeyEqual, Allocator>& x);

// accessors:

key_equal key_eq() const;
hasher hash_funct() const;
iterator begin() const;
iterator end() const;
bool empty() const;
size_type size() const;
size_type max_size() const;

// insert/erase

pair<iterator, bool> insert(const value_type& x);
template <class InputIterator>
void insert(InputIterator first, InputIterator last);
void erase(iterator position);
size_type erase(const key_type& x);
void erase(iterator first, iterator last);

// search operations:

iterator find(const key_type& x) const;
size_type count(const key_type& x) const;
pair<iterator, iterator> equal_range(const key_type& x) const;

// hash table size operations

size_type bucket_count() const;
void resize(size_type buckets);
};

template <class Key, class Hasher=hash<Key>, class KeyEqual = equal_to<Key>,
template <class U> class Allocator = allocator>
class hash_multiset {
public:

// typedefs:

typedef Key key_type;
typedef Key value_type;
typedef Hasher hasher;
typedef KeyEqual key_equal;

iterator is a constant forward iterator referring to const value_type. The exact type is implementation dependent and determined by Allocator.

const_iterator is the same type as iterator.

2.2.3 Hash_multiset

hash_multiset is a kind of hash table that supports equal keys (possibly contains multiple copies of the same key value) and provides for fast retrieval of the keys themselves.

template <class Key, class Hasher=hash<Key>, class KeyEqual = equal_to<Key>,
template <class U> class Allocator = allocator>
class hash_multiset {
public:

// typedefs:
typedef pointer;
typedef reference;
typedef const_reference;
typedef iterator;
typedef iterator const_iterator;
typedef size_type;
typedef difference_type;

// allocation/deallocation

hash_multiset(size_type table_size = 1007,
              const Hasher& hf = Hasher(),
              const KeyEqual& equal = KeyEqual());
template <class InputIterator>
hash_multiset(InputIterator first, InputIterator last,
              size_type table_size = 10007,
              const Hasher& hf = Hasher(),
              const KeyEqual& equal = KeyEqual());
~hash_multiset();

// copying, assignment, swap

hash_multiset(const hash_multiset<Key, Hasher, KeyEqual, Allocator>& x);
hash_multiset<Key, Hasher, KeyEqual, Allocator>&
operator=(const hash_multiset<Key, Hasher, KeyEqual, Allocator>& x);
void swap(hash_multiset<Key, Hasher, KeyEqual, Allocator>& x);

// accessors:

key_equal key_eq() const;
hasher hash_funct() const;
iterator begin() const;
iterator end() const;
bool empty() const;
size_type size() const;
size_type max_size() const;

// insert/erase

iterator insert(const value_type& x);
template <class InputIterator>
void insert(InputIterator first, InputIterator last);
void erase(iterator position);
size_type erase(const key_type& x);
void erase(iterator first, iterator last);

// search operations:

iterator find(const key_type& x) const;
size_type count(const key_type& x) const;
pair<iterator, iterator> equal_range(const key_type& x) const;

// hash table size operations

size_type bucket_count() const;
void resize(size_type buckets);
template <class Key, class Hasher, class KeyEqual, Allocator>
bool operator==(const hash_multiset<Key, Hasher, KeyEqual, Allocator>& x,
               const hash_multiset<Key, Hasher, KeyEqual, Allocator>& y);

iterator is a constant forward iterator referring to const value_type. The exact type is implementation dependent and determined by Allocator.

const_iterator is the same type as iterator.

size_type is an unsigned integral type. The exact type is implementation dependent and determined by Allocator.

difference_type is a signed integral type. The exact type is implementation dependent and determined by Allocator.

2.2.4 Hash_map

hash_map is a kind of hash table that supports unique keys (contains at most one of each key value) and provides for fast retrieval of values of another type T based on the keys.

hash_map<Key,T,Hasher,KeyEqual, Allocator> { 
  public:

    // typedefs:

typedef Key key_type;
    typedef pair<const Key, T> value_type;
    typedef Hasher hasher;
    typedef KeyEqual key_equal;
    typedef pointer;
    typedef reference;
    typedef const_reference;
    typedef iterator;
    typedef const_iterator;
    typedef size_type;
    typedef difference_type;

    // allocation/deallocation

    hash_map(size_type table_size = 1007,
             const Hasher& hf = Hasher(),
             const KeyEqual& equal = KeyEqual());
    template <class InputIterator>
    hash_map(InputIterator first, InputIterator last,
             size_type table_size = 10007,
             const Hasher& hf = Hasher());
    ~hash_map();

    // copying, assignment, swap

    hash_map(const hash_map<Key, T, Hasher, KeyEqual, Allocator>& x);
    hash_map<Key,T,Hasher,KeyEqual, Allocator>&
      operator=(const hash_map<Key, T, Hasher, KeyEqual, Allocator>& x);
    void swap(hash_map<Key, T, Hasher, KeyEqual, Allocator>& x);
// accessors:

key_equal key_eq() const;
hasher hash_funct() const;
iterator begin();
const_iterator begin() const;
const_iterator end();
const_iterator end() const;
bool empty() const;
size_type size() const;
size_type max_size() const;
Allocator<T>::reference operator[](const key_type& k);

// insert/erase

pair<iterator, bool> insert(const value_type& x);
template <class InputIterator>
void insert(InputIterator first, InputIterator last);
void erase(iterator position);
size_type erase(const key_type& x);
void erase(iterator first, iterator last);

// search operations:

iterator find(const key_type& x);
const_iterator find(const key_type& x) const;
size_type count(const key_type& x) const;
pair<iterator, iterator> equal_range(const key_type& x);
pair<const_iterator, const_iterator> equal_range(const key_type& x) const;

// hash table size operations

size_type bucket_count() const;
void resize(size_type buckets);

};

template <class Key, class T, class Hasher, class KeyEqual, Allocator>
bool operator==(const hash_map<Key, T, Hasher, KeyEqual, Allocator>& x,
const hash_map<Key, T, Hasher, KeyEqual, Allocator>& y);

iterator is a forward iterator referring to value_type. The exact type is implementation dependent and
determined by Allocator.

const_iterator is a constant forward iterator referring to const value_type. The exact type is
implementation dependent and determined by Allocator. It is guaranteed that there is a constructor for
const_iterator out of iterator.

size_type is an unsigned integral type. The exact type is implementation dependent and determined by
Allocator.

difference_type is a signed integral type. The exact type is implementation dependent and determined by
Allocator.

In addition to the standard set of member functions of hash tables, hash_map provides
Allocator<T>::reference operator[](const key_type&). For a hash_map m and key k, m[k] is
semantically equivalent to(*(m.insert(make_pair(k, T()))).first)).second.
2.2.5 Hash_multimap

hash_multimap is a kind of hash table that supports equal keys (possibly contains multiple copies of the same key value) and provides for fast retrieval of values of another type T based on the keys.

template <class Key, class Hasher=hash<Key>, class KeyEqual = equal_to<Key>,
          template <class U> class Allocator = allocator>
class hash_multimap {
public:

   // typedefs:
   typedef Key key_type;
   typedef pair<const Key, T> value_type;
   typedef Hasher hasher;
   typedef KeyEqual key_equal;
   typedef pointer;
   typedef reference;
   typedef const_reference;
   typedef iterator;
   typedef const_iterator;
   typedef size_type;
   typedef difference_type;

   // allocation/deallocation
   hash_multimap(size_type table_size = 1007,
                 const Hasher& hf = Hasher(),
                 const KeyEqual& equal = KeyEqual());
   template <class InputIterator>
   hash_multimap(InputIterator first, InputIterator last,
                 size_type table_size = 10007,
                 const Hasher& hf = Hasher(),
                 const KeyEqual& equal = KeyEqual());
   ~hash_multimap();

   // copying, assignment, swap
   hash_multimap(const hash_multimap<Key, T, Hasher, KeyEqual, Allocator>& x);
   hash_multimap<Key, T, Hasher, KeyEqual, Allocator>&
            operator=(const hash_multimap<Key, T, Hasher, KeyEqual, Allocator>& x);
   void swap(hash_multimap<Key, T, Hasher, KeyEqual, Allocator>& x);

   // accessors:
   key_equal key_eq() const;
   hasher hash_funct() const;
   iterator begin();
   const_iterator begin() const;
   iterator end();
   const_iterator end() const;
   bool empty() const;
   size_type size() const;
   size_type max_size() const;

   // insert/erase
   iterator insert(const value_type& x);
template <class InputIterator>
    void insert(InputIterator first, InputIterator last);
    void erase(iterator position);
    size_type erase(const key_type& x);
    void erase(iterator first, iterator last);

    // search operations:
    iterator find(const key_type& x);
    const_iterator find(const key_type& x) const;
    size_type count(const key_type& x) const;
    pair<iterator, iterator> equal_range(const key_type& x);
    pair<const_iterator, const_iterator> equal_range(const key_type& x) const;

    // hash table size operations
    size_type bucket_count() const;
    void resize(size_type size);
    
    template <class Key, class T, class Hasher, class KeyEqual, Allocator>
    bool operator==(const hash_multimap<Key, T, Hasher, KeyEqual, Allocator>& x,
                    const hash_multimap<Key, T, Hasher, KeyEqual, Allocator>& y);

iterator is a forward iterator referring to value_type. The exact type is implementation dependent and determined by Allocator.

const_iterator is the a constant forward iterator referring to const value_type. The exact type is implementation dependent and determined by Allocator. It is guaranteed that there is a constructor for const_iterator out of iterator.

size_type is an unsigned integral type. The exact type is implementation dependent and determined by Allocator.

difference_type is a signed integral type. The exact type is implementation dependent and determined by Allocator.

3 Hash Table Implementations

We have developed two separate reference hash table implementations that meet all of the above requirements. Both use the method of separate chaining, but with somewhat different ways of storing lists in the hash buckets. The major difference between the two implementations is the way they handle expansion. More detail about these implementations, including performance comparisons, will be given in a separate report.

In order to realize the amortized constant time bounds for insertion, searching, and erasure that are required for hash tables (Time Complexity entries in Table 2), it is necessary that hash tables expand dynamically (increasing the number of buckets) as the number of items stored grows. A simple method of expansion is often used: allocate another, larger, table of buckets; rehash all items in the old table into the new table; and deallocate the old table. This method, which we call intermittent resizing, it can be very time-consuming, perhaps prohibitively so in some applications requiring very fast response-times. A different implementation avoids this potential bottleneck by expanding (or contracting) the table on a gradual basis; it could be called gradual resizing. The method used is based mainly on one described in Per-Ake Larson, CACM, Vol. 31, Number 4, April 1988.
4 Acknowledgements

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