Information technology — Programming languages, their environments, and system software interfaces — Floating-point extensions for C — Part 4: Supplementary functions

Warning
This document is not an ISO International Standard. It is distributed for review and comment. It is subject to change without notice and may not be referred to as an International Standard.

Recipients of this draft are invited to submit, with their comments, notification of any relevant patent rights of which they are aware and to provide supporting documentation.
Copyright notice

This ISO document is a working draft or committee draft and is copyright-protected by ISO. While the reproduction of working drafts or committee drafts in any form for use by participants in the ISO standards development process is permitted without prior permission from ISO, neither this document nor any extract from it may be reproduced, stored or transmitted in any form for any other purpose without prior written permission from ISO.

Requests for permission to reproduce this document for the purpose of selling it should be addressed as shown below or to ISO’s member body in the country of the requester:

ISO copyright office
Case postale 56 CH-1211 Geneva 20
Tel. +41 22 749 01 11
Fax +41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Reproduction for sales purposes may be subject to royalty payments or a licensing agreement.

Violators may be prosecuted.
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>v</td>
</tr>
<tr>
<td>Background</td>
<td>v</td>
</tr>
<tr>
<td>IEC 60559 floating-point standard</td>
<td>v</td>
</tr>
<tr>
<td>C support for IEC 60559</td>
<td>vi</td>
</tr>
<tr>
<td>Purpose</td>
<td>vii</td>
</tr>
<tr>
<td>Additional background on supplementary functions</td>
<td>vii</td>
</tr>
<tr>
<td>1. Scope</td>
<td>1</td>
</tr>
<tr>
<td>2. Conformance</td>
<td>1</td>
</tr>
<tr>
<td>3. Normative references</td>
<td>1</td>
</tr>
<tr>
<td>4. Terms and definitions</td>
<td>2</td>
</tr>
<tr>
<td>5. C standard conformance</td>
<td>2</td>
</tr>
<tr>
<td>5.1 Freestanding implementations</td>
<td>2</td>
</tr>
<tr>
<td>5.2 Predefined macros</td>
<td>2</td>
</tr>
<tr>
<td>5.3 Standard headers</td>
<td>2</td>
</tr>
<tr>
<td>6. Operation binding</td>
<td>4</td>
</tr>
<tr>
<td>7. Mathematical functions in <code>&lt;math.h&gt;</code></td>
<td>5</td>
</tr>
<tr>
<td>8. Reduction functions in <code>&lt;math.h&gt;</code></td>
<td>17</td>
</tr>
<tr>
<td>9. Future directions for <code>&lt;complex.h&gt;</code></td>
<td>23</td>
</tr>
<tr>
<td>10. Type-generic macros <code>&lt;tgmath.h&gt;</code></td>
<td>23</td>
</tr>
<tr>
<td>Bibliography</td>
<td>25</td>
</tr>
</tbody>
</table>
Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work. In the field of information technology, ISO and IEC have established a joint technical committee, ISO/IEC JTC 1.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO and IEC shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/IEC JTC 1, Information technology, SC 22, Programming languages, their environments, and system software interfaces.

ISO/IEC TS 18661 consists of the following parts, under the general title Information technology—Programming languages, their environments, and system software interfaces — Floating-point extensions for C:

— Part 1: Binary floating-point arithmetic
— Part 2: Decimal floating-point arithmetic
— Part 3: Interchange and extended types
— Part 4: Supplementary functions
— Part 5: Supplementary attributes


Part 2 supersedes ISO/IEC TR 24732:2009, Information technology — Programming languages, their environments and system software interfaces — Extension for the programming language C to support decimal floating-point arithmetic.

Introduction

Background

IEC 60559 floating-point standard

The IEEE 754-1985 standard for binary floating-point arithmetic was motivated by an expanding diversity in floating-point data representation and arithmetic, which made writing robust programs, debugging, and moving programs between systems exceedingly difficult. Now the great majority of systems provide data formats and arithmetic operations according to this standard. The IEC 60559:1989 international standard was equivalent to the IEEE 754-1985 standard. Its stated goals were:

1 Facilitate movement of existing programs from diverse computers to those that adhere to this standard.

2 Enhance the capabilities and safety available to programmers who, though not expert in numerical methods, may well be attempting to produce numerically sophisticated programs. However, we recognize that utility and safety are sometimes antagonists.

3 Encourage experts to develop and distribute robust and efficient numerical programs that are portable, by way of minor editing and recompilation, onto any computer that conforms to this standard and possesses adequate capacity. When restricted to a declared subset of the standard, these programs should produce identical results on all conforming systems.

4 Provide direct support for
   a. Execution-time diagnosis of anomalies
   b. Smoother handling of exceptions
   c. Interval arithmetic at a reasonable cost

5 Provide for development of
   a. Standard elementary functions such as exp and cos
   b. Very high precision (multiword) arithmetic
   c. Coupling of numerical and symbolic algebraic computation

6 Enable rather than preclude further refinements and extensions.

To these ends, the standard specified a floating-point model comprising:

-formats – for binary floating-point data, including representations for Not-a-Number (NaN) and signed infinities and zeros

-operations – basic arithmetic operations (addition, multiplication, etc.) on the format data to compose a well-defined, closed arithmetic system; also specified conversions between floating-point formats and decimal character sequences, and a few auxiliary operations

-context – status flags for detecting exceptional conditions (invalid operation, division by zero, overflow, underflow, and inexact) and controls for choosing different rounding methods

The ISO/IEC/IEEE 60559:2011 international standard is equivalent to the IEEE 754-2008 standard for floating-point arithmetic, which is a major revision to IEEE 754-1985.

The revised standard specifies more formats, including decimal as well as binary. It adds a 128-bit binary format to its basic formats. It defines extended formats for all of its basic formats. It specifies data interchange
formats (which may or may not be arithmetic), including a 16-bit binary format and an unbounded tower of wider formats. To conform to the floating-point standard, an implementation must provide at least one of the basic formats, along with the required operations.

The revised standard specifies more operations. New requirements include – among others – arithmetic operations that round their result to a narrower format than the operands (with just one rounding), more conversions with integer types, more classifications and comparisons, and more operations for managing flags and modes. New recommendations include an extensive set of mathematical functions and seven reduction functions for sums and scaled products.

The revised standard places more emphasis on reproducible results, which is reflected in its standardization of more operations. For the most part, behaviors are completely specified. The standard requires conversions between floating-point formats and decimal character sequences to be correctly rounded for at least three more decimal digits than is required to distinguish all numbers in the widest supported binary format; it fully specifies conversions involving any number of decimal digits. It recommends that transcendental functions be correctly rounded.

The revised standard requires a way to specify a constant rounding direction for a static portion of code, with details left to programming language standards. This feature potentially allows rounding control without incurring the overhead of runtime access to a global (or thread) rounding mode.

Other features recommended by the revised standard include alternate methods for exception handling, controls for expression evaluation (allowing or disallowing various optimizations), support for fully reproducible results, and support for program debugging.

The revised standard, like its predecessor, defines its model of floating-point arithmetic in the abstract. It neither defines the way in which operations are expressed (which might vary depending on the computer language or other interface being used), nor does it define the concrete representation (specific layout in storage, or in a processor's register, for example) of data or context, except that it does define specific encodings that are to be used for data that may be exchanged between different implementations that conform to the specification.

IEC 60559 does not include bindings of its floating-point model for particular programming languages. However, the revised standard does include guidance for programming language standards, in recognition of the fact that features of the floating-point standard, even if well supported in the hardware, are not available to users unless the programming language provides a commensurate level of support. The implementation's combination of both hardware and software determines conformance to the floating-point standard.

C support for IEC 60559

The C standard specifies floating-point arithmetic using an abstract model. The representation of a floating-point number is specified in an abstract form where the constituent components (sign, exponent, significand) of the representation are defined but not the internals of these components. In particular, the exponent range, significand size, and the base (or radix) are implementation-defined. This allows flexibility for an implementation to take advantage of its underlying hardware architecture. Furthermore, certain behaviors of operations are also implementation-defined, for example in the area of handling of special numbers and in exceptions.

The reason for this approach is historical. At the time when C was first standardized, before the floating-point standard was established, there were various hardware implementations of floating-point arithmetic in common use. Specifying the exact details of a representation would have made most of the existing implementations at the time not conforming.


**Purpose**

The purpose of ISO/IEC TS 18661 is to provide a C language binding for ISO/IEC/IEEE 60559:2011, based on the C11 standard, that delivers the goals of ISO/IEC/IEEE 60559 to users and is feasible to implement. It is organized into five Parts.

Part 1 provides changes to C11 that cover all the requirements, plus some basic recommendations, of ISO/IEC/IEEE 60559:2011 for binary floating-point arithmetic. C implementations intending to support ISO/IEC/IEEE 60559:2011 are expected to conform to conditionally normative Annex F as enhanced by the changes in Part 1.

Part 2 enhances ISO/IEC TR 24732 to cover all the requirements, plus some basic recommendations, of ISO/IEC/IEEE 60559:2011 for decimal floating-point arithmetic. C implementations intending to provide an extension for decimal floating-point arithmetic supporting ISO/IEC/IEEE 60559:2011 are expected to conform to Part 2.

Part 3 specifies types and other support for interchange and extended formats recommended in ISO/IEC/IEEE 60559:2011. C implementations intending to provide an extension for these formats are expected to conform to Part 3.

Part 4, this document, specifies functions for operations recommended in ISO/IEC/IEEE 60559:2011. C implementations intending to provide an extension for these operations are expected to conform to Part 4.

Part 5 specifies support for attributes recommended in ISO/IEC/IEEE 60559:2011. C implementations intending to provide an extension for these attributes are expected to conform to Part 5.

**Additional background on supplementary functions**

This document uses the term supplementary functions to refer to functions that provide operations recommended, but not required, by IEC 60559.

ISO/IEC/IEEE 60559:2011 specifies and recommends a more extensive set of mathematical operations than C11 provides. The IEC 60559 specification is generally consistent with C11, though it adds requirements for symmetry and antisymmetry. This part of ISO/IEC TS 18661 extends the specification in Library subclause 7.12 Mathematics to include the complete set of IEC 60559 mathematical operations. For implementations conforming to Annex F, it also requires full IEC 60559 semantics, including symmetry and antisymmetry properties.

IEC 60559 requires correct rounding for its required operations (squareRoot, fusedMultiplyAdd, etc.), and recommends correct rounding for its recommended mathematical operations. This part of ISO/IEC TS 18661 reserves identifiers, with cr prefixes, for C functions corresponding to correctly rounded versions of the IEC 60559 mathematical operations, which may be provided at the option of the implementation. For example, the identifier crexp is reserved for a correctly rounded version of the exp function.

IEC 60559 also specifies and recommends reduction operations, which operate on vector operands. These operations, which compute sums and products, may associate in any order and may evaluate in any wider format. Hence, unlike other IEC 60559 operations, they do not have unique specified results. This part of ISO/IEC TS 18661 extends the specification in Library subclause 7.12 Mathematics to include functions corresponding to the IEC 60559 reduction operations. For implementations conforming to Annex F, it also requires the IEC 60559 specified behavior for floating-point exceptions.
Information technology — Programming languages, their environments, and system software interfaces — Floating-point extensions for C —

Part 4:
Supplementary functions

1 Scope

This part of ISO/IEC TS 18661 extends programming language C to include functions specified and recommended in ISO/IEC/IEEE 60559:2011.

2 Conformance

An implementation conforms to this part of ISO/IEC TS 18661 if

a) It meets the requirements for a conforming implementation of C11 with all the changes to C11 as specified in parts 1-4 of ISO/IEC TS 18661;

b) It conforms to ISO/IEC TS 18661-1 or ISO/IEC TS 18661-2 (or both); and

c) It defines \texttt{__STDC_IEC_60559_FUNCS__} to \texttt{201\_ymnL}.

3 Normative references

The following referenced documents are indispensable for the application of this document. Only the editions cited apply.

ISO/IEC 9899:2011, Information technology — Programming languages, their environments and system software interfaces — Programming Language C

ISO/IEC 9899:2011/Cor.1:2012, Technical Corrigendum 1


ISO/IEC 18661-2:yyyy, Information Technology — Programming languages, their environments, and system software interfaces — Floating-point extensions for C — Part 2: Decimal floating-point arithmetic

ISO/IEC 18661-3:yyyy, Information Technology — Programming languages, their environments, and system software interfaces — Floating-point extensions for C — Part 3: Interchange and extended types

Changes specified in this part of ISO/IEC TS 18661 are relative to ISO/IEC 9899:2011, including Technical Corrigendum 1 (ISO/IEC 9899:2011/Cor. 1:2012), together with the changes from parts 1-3 of ISO/IEC TS 18661.
4 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC 9899:2011 and ISO/IEC/IEEE 60559:2011 and the following apply.


5 C standard conformance

5.1 Freestanding implementations

The specification in C11 + TS18661-1 + TS18661-2 + TS18661-3 allows freestanding implementations to conform to this Part of Technical Specification 18661.

5.2 Predefined macros

Change to C11 + TS18661-1 + TS18661-2 + TS18661-3:

In 6.10.8.3#1, add:

```
__STDC_IEC_60559_FUNCS__ The integer constant 201ymmL, intended to indicate support of functions specified and recommended in IEC 60559.
```

5.3 Standard headers

The new identifiers added to C11 library headers by this Part of Technical Specification 18661 are defined or declared by their respective headers only if __STDC_WANT_IEC_60559_FUNCS_EXT__ is defined as a macro at the point in the source file where the appropriate header is first included. The following changes to C11 + TS18661-1 + TS18661-2 + TS18661-3 list these identifiers in each applicable library subclause.

Changes to C11 + TS18661-1 + TS18661-2 + TS18661-3:

In 7.12, renumber paragraph 1e to 1h, and after paragraph 1d insert the paragraphs:

```
[1e] The following identifiers are declared only if __STDC_WANT_IEC_60559_FUNCS_EXT__ is defined as a macro at the point in the source file where <math.h> is first included:

```

```c
exp2m1 rootnf sinpil
exp2m1f rootnl tanpi
exp2m1l pownf tanpif
exp10 pownl reduc_sum
exp10f powr reduc_sumf
tanpil
exp10l powrf reduc_suml
exp10m1 powrl reduc_sumabs
exp10m1f acospif reduc_sumabsf
acospil
logpl acospifs reduc_sumabsf1
logpif acospil reduc_sumsq
logplf asinpil reduc_sumsqf
log2p1f asinpil reduc_sumsql
40
log2p1f asinpil reduc_sumprod
log2p1l atanpi reduc_sumprod1
log10p1 atanpif scaled_prod
log10p1f atanpil
```

© ISO/IEC 2014 – All rights reserved
The following identifiers are declared only if __STDC_WANT_IEC_60559_DFP_EXT__ and __STDC_WANT_IEC_60559_FUNCS_EXT__ are defined as macros at the point in the source file where <math.h> is first included:

for supported types _DecimalN, where N = 32, 64, and 128:

\[
\begin{align*}
\text{exp2m1d}N & \quad \text{pownd}N & \quad \text{tanpid}N \\
\text{exp10d}N & \quad \text{powrd}N & \quad \text{reduc_sumd}N \\
\text{exp10mld}N & \quad \text{acospid}N & \quad \text{reduc_sumab}dN \\
\text{logp1d}N & \quad \text{asinpid}N & \quad \text{reduc_sumqs}qN \\
\text{log2p1d}N & \quad \text{atanpid}N & \quad \text{reduc_sumprod}dN \\
\text{log10p1d}N & \quad \text{atan2p1d}N & \quad \text{scaled_prodd}N \\
\text{rsqrt}dN & \quad \text{cospid}N & \quad \text{scaled_prods}umd}N \\
\text{compound}ndN & \quad \text{sinpid}N & \quad \text{scaled_proddiff}d}N \\
\text{root}nN
\end{align*}
\]

[1f] For supported types _FloatN:

\[
\begin{align*}
\text{exp2m1f}N & \quad \text{pownf}N & \quad \text{tanpif}N \\
\text{exp10f}N & \quad \text{powrf}N & \quad \text{reduc_sumf}N \\
\text{exp10m1f}N & \quad \text{acospif}N & \quad \text{reduc_sumabsf}N \\
\text{logp1f}N & \quad \text{asinpif}N & \quad \text{reduc_sumqs}fN \\
\text{log2p1f}N & \quad \text{atanpif}N & \quad \text{reduc_sumprod}fN \\
\text{log10p1f}N & \quad \text{atan2p1f}N & \quad \text{scaled_prodf}N \\
\text{rsqrtf}N & \quad \text{cospif}N & \quad \text{scaled_prods}umf}N \\
\text{compound}nfN & \quad \text{sinpif}N & \quad \text{scaled_proddiff}f}N \\
\text{root}nfN
\end{align*}
\]

[1g] For supported types _FloatNx:

\[
\begin{align*}
\text{exp2m1f Nx} & \quad \text{pownf Nx} & \quad \text{tanpif Nx} \\
\text{exp10f Nx} & \quad \text{powrf Nx} & \quad \text{reduc_sumfNx} \\
\text{exp10m1f Nx} & \quad \text{acospif Nx} & \quad \text{reduc_sumabsfNx} \\
\text{logp1f Nx} & \quad \text{asinpif Nx} & \quad \text{reduc_sumqs}fNx \\
\text{log2p1f Nx} & \quad \text{atanpif Nx} & \quad \text{reduc_sumprod}fNx \\
\text{log10p1f Nx} & \quad \text{atan2p1f Nx} & \quad \text{scaled_prodfNx} \\
\text{rsqrtf}Nx & \quad \text{cospif Nx} & \quad \text{scaled_prods}umf}Nx \\
\text{compound}nfNx & \quad \text{sinpif Nx} & \quad \text{scaled_proddiff}fNx \\
\text{root}nfNx
\end{align*}
\]
for supported types \texttt{\_Decimal\textbackslash N}, where \( N \neq 32, 64, \) and 128:

\begin{align*}
\exp_{2m1} & \quad \text{pown}_{\text{\_Decimal\textbackslash N}} & \quad \tan_{\text{\_Decimal\textbackslash N}} \\
\exp_{10} & \quad \text{pown}_{\text{\_Decimal\textbackslash N}} & \quad \text{reduc}_{\text{sum}}_{\text{\_Decimal\textbackslash N}} \\
\exp_{10m1} & \quad \text{acosp}_{\text{\_Decimal\textbackslash N}} & \quad \text{reduc}_{\text{sumabs}}_{\text{\_Decimal\textbackslash N}} \\
\log_{1} & \quad \text{acosp}_{\text{\_Decimal\textbackslash N}} & \quad \text{reduc}_{\text{sumsqd}}_{\text{\_Decimal\textbackslash N}} \\
\log_{2} & \quad \text{atan}_{\text{\_Decimal\textbackslash N}} & \quad \text{reduc}_{\text{sumprodd}}_{\text{\_Decimal\textbackslash N}} \\
\log_{10} & \quad \text{atan}_{2p1}_{\text{\_Decimal\textbackslash N}} & \quad \text{scaled}_{\text{prodd}}_{\text{\_Decimal\textbackslash N}} \\
\rsqrt & \quad \text{cos}_{\text{\_Decimal\textbackslash N}} & \quad \text{scaled}_{\text{prodsqd}}_{\text{\_Decimal\textbackslash N}} \\
\text{compound}_{\text{\_Decimal\textbackslash N}} & \quad \text{sin}_{\text{\_Decimal\textbackslash N}} & \quad \text{scaled}_{\text{proddiffd}}_{\text{\_Decimal\textbackslash N}} \\
\rootn & \quad \text{sin}_{\text{\_Decimal\textbackslash N}} & \quad \text{scaled}_{\text{proddiffd}}_{\text{\_Decimal\textbackslash N}}
\end{align*}

for supported types \texttt{\_Decimal\textbackslash N\textbackslash x}:

\begin{align*}
\exp_{2m1} & \quad \text{pown}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \tan_{\text{\_Decimal\textbackslash N\textbackslash x}} \\
\exp_{10} & \quad \text{pown}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{reduc}_{\text{sum}}_{\text{\_Decimal\textbackslash N\textbackslash x}} \\
\exp_{10m1} & \quad \text{acosp}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{reduc}_{\text{sumabs}}_{\text{\_Decimal\textbackslash N\textbackslash x}} \\
\log_{1} & \quad \text{acosp}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{reduc}_{\text{sumsqd}}_{\text{\_Decimal\textbackslash N\textbackslash x}} \\
\log_{2} & \quad \text{atan}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{reduc}_{\text{sumprodd}}_{\text{\_Decimal\textbackslash N\textbackslash x}} \\
\log_{10} & \quad \text{atan}_{2p1}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{scaled}_{\text{prodd}}_{\text{\_Decimal\textbackslash N\textbackslash x}} \\
\rsqrt & \quad \text{cos}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{scaled}_{\text{prodsqd}}_{\text{\_Decimal\textbackslash N\textbackslash x}} \\
\text{compound}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{sin}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{scaled}_{\text{proddiffd}}_{\text{\_Decimal\textbackslash N\textbackslash x}} \\
\rootn & \quad \text{sin}_{\text{\_Decimal\textbackslash N\textbackslash x}} & \quad \text{scaled}_{\text{proddiffd}}_{\text{\_Decimal\textbackslash N\textbackslash x}}
\end{align*}

After 7.25#1c, insert the paragraph:

[1d] The following identifiers are defined as type-generic macros only if \texttt{\_STDC_WANT_IEC_60559\_FUNCS\_EXT\_} is defined as a macro at the point in the source file where <tgmath.h> is first included:

\begin{align*}
\exp_{2ml} & \quad \text{rsqrt} & \quad \text{asinpi} \\
\exp_{10} & \quad \text{compound}_{\text{\_Decimal\textbackslash N}} & \quad \text{atanpi} \\
\exp_{10ml} & \quad \text{rootn} & \quad \text{atan}_{2p1} \\
\log_{1} & \quad \text{pown} & \quad \text{cos}_{\text{\_Decimal\textbackslash N}} \\
\log_{2} & \quad \text{powr} & \quad \text{sin}_{\text{\_Decimal\textbackslash N}} \\
\log_{10} & \quad \text{acosp}_{\text{\_Decimal\textbackslash N}} & \quad \tan_{\text{\_Decimal\textbackslash N}}
\end{align*}

6 Operation binding

The following change to C11 + TS18661-1 + TS18661-2 + TS18661-3 shows how functions in C11 and in this Part of Technical Specification 18661 provide operations recommended in IEC 60559.

Change to C11 + TS18661-1 + TS18661-2 + TS18661-3:

After F.3#22, add:

[23] The C functions in the following table provide operations recommended by IEC 60559 and similar operations. Correct rounding, which IEC 60559 specifies for its operations (except for the reduction operations), is not required for the C functions in the table. See also 7.31.6a.

\begin{table}[h]
\small
\begin{tabular}{|c|c|c|}
\hline
\textbf{IEC 60559 operation} & \textbf{C function} & \textbf{Clauses - C11} \\
\hline
\text{exp} & \text{exp} & \text{7.12.6.1, F.10.3.1} \\
\text{exp}_{m1} & \text{exp}_{m1} & \text{7.12.6.3, F.10.3.3} \\
\text{exp}_2 & \text{exp}_2 & \text{7.12.6.2, F.10.3.2} \\
\text{exp}_{2m1} & \text{exp}_{2m1} & \text{7.12.6.14, F.10.3.14} \\
\text{exp}_{10} & \text{exp}_{10} & \text{7.12.6.15, F.10.3.15} \\
\text{exp}_{10m1} & \text{exp}_{10m1} & \text{7.12.6.16, F.10.3.16} \\
\hline
\end{tabular}
\end{table}
7 Mathematical functions in `<math.h>`

This clause specifies changes to C11 + TS18661-1 + TS18661-2 + TS18661-3 to include functions that support mathematical operations recommended by IEC 60559. The changes reserve names for correctly rounded versions of the functions. IEC 60559 recommends support for the correctly rounded functions. The changes also include support for the symmetry and antisymmetry properties that IEC 60559 specifies for mathematical functions.
Changes to C11 + TS18661-1 + TS18661-2 + TS18661-3:

After 7.12.4.7, insert the following:

7.12.4.8 The \texttt{acospi} functions

Synopsis

[1] \texttt{
#include <math.h>

double acospi(double x);
float acospif(float x);
long double acospil(long double x);
_FloatN acospifN(_FloatN x);
_FloatNx acospifNx(_FloatNx x);
_DecimalN acospilN(_DecimalN x);
_DecimalNx acospilNx(_DecimalNx x);
}

Description

[2] The \texttt{acospi} functions compute the principal value of the arc cosine of \( x \), divided by \( \pi \), thus measuring the angle in half-revolutions. A domain error occurs for arguments not in the interval \([-1, +1]\).

Returns

[3] The \texttt{acospi} functions return \( \arccos(x) / \pi \), in the interval \([0, 1]\).

7.12.4.9 The \texttt{asinpi} functions

Synopsis

[1] \texttt{
#include <math.h>

double asinpi(double x);
float aminpif(float x);
long double asinpl(long double x);
_FloatN asinpfN(_FloatN x);
_FloatNx asinpfNx(_FloatNx x);
_DecimalN asinpilN(_DecimalN x);
_DecimalNx asinpilNx(_DecimalNx x);
}

Description

[2] The \texttt{asinpi} functions compute the principal value of the arc sine of \( x \), divided by \( \pi \), thus measuring the angle in half-revolutions. A domain error occurs for arguments not in the interval \([-1, +1]\). A range error occurs if the magnitude of nonzero \( x \) is too small.

Returns

[3] The \texttt{asinpi} functions return \( \arcsin(x) / \pi \), in the interval \([-1/2, +1/2]\).
7.12.4.10 The atanpi functions

Synopsis

[1] #include <math.h>
   double atanpi(double x);
   float atanpif(float x);
   long double atanpil(long double x);
   _FloatN atanpifN(_FloatN x);
   _FloatNx atanpifNx(_FloatNx x);
   _DecimalN atanpilN(_DecimalN x);
   _DecimalNx atanpilNx(_DecimalNx x);

Description

[2] The atanpi functions compute the principal value of the arc tangent of x, divided by π, thus measuring the angle in half-revolutions. A range error occurs if the magnitude of nonzero x is too small.

Returns

[3] The atanpi functions return arctan(x) / π, in the interval [-1/2, +1/2].

7.12.4.11 The atan2pi functions

Synopsis

[1] #include <math.h>
   double atan2pi(double y, double x);
   float atan2pif(float y, float x);
   long double atan2pil(long double y, long double x);
   _FloatN atan2pifN(_FloatN y, _FloatN x);
   _FloatNx atan2pifNx(_FloatNx y, _FloatNx x);
   _DecimalN atan2pilN(_DecimalN y, _DecimalN x);
   _DecimalNx atan2pilNx(_DecimalNx y, _DecimalNx x);

Description

[2] The atan2pi functions compute the angle, measured in half-revolutions, subtended at the origin by the point (x, y) and the positive x-axis. Thus, atan2pi computes arctan(y/x) / π, in the range [-1, +1]. A domain error may occur if both arguments are zero. A range error occurs if x is positive and the magnitude of nonzero y/x is too small.

Returns

[3] The atan2pi functions return the computed angle, in the interval [-1, +1].
7.12.4.12 The \texttt{cospi} functions

Synopsis

[1] 

\begin{verbatim}
#include <math.h>

double cospi(double x);
float cospif(float x);
long double cospil(long double x);
_FloatN cospifN(_FloatN x);
_FloatNx cospifNx(_Float Nx x);
.DecimalN cospidN(_DecimalN x);
_DecimalNx cospidNx(_DecimalNx x);
\end{verbatim}

Description

[2] The \texttt{cospi} functions compute the cosine of $\pi \times x$, thus regarding $x$ as a measurement in half-revolutions.

Returns

[3] The \texttt{cospi} functions return $\cos(\pi \times x)$.

7.12.4.13 The \texttt{sinpi} functions

Synopsis

[1] 

\begin{verbatim}
#include <math.h>

double sinpi(double x);
float sinpif(float x);
long double sinpil(long double x);
_FloatN sinpifN(_FloatN x);
_FloatNx sinpifNx(_Float Nx x);
.DecimalN sinpidN(_DecimalN x);
_DecimalNx sinpidNx(_DecimalNx x);
\end{verbatim}

Description

[2] The \texttt{sinpi} functions compute the sine of $\pi \times x$, thus regarding $x$ as a measurement in half-revolutions.

Returns

[3] The \texttt{sinpi} functions return $\sin(\pi \times x)$.

7.12.4.14 The \texttt{tanpi} functions

Synopsis

[1] 

\begin{verbatim}
#include <math.h>

double tanpi(double x);
float tanpif(float x);
long double tanpil(long double x);
_FloatN tanpifN(_FloatN x);
_FloatNx tanpifNx(_Float Nx x);
.DecimalN tanpidN(_DecimalN x);
_DecimalNx tanpidNx(_DecimalNx x);
\end{verbatim}
Description

[2] The tan\(\pi\) functions compute the tangent of \(\pi \times x\), thus regarding \(x\) as a measurement in half-revolutions. A pole error may occur for arguments \(n + 1/2\), for integers \(n\).

Returns

[3] The tan\(\pi\) functions return \(\tan(\pi \times x)\).

In 7.12.6.9, replace the subclause title:

7.12.6.9 The log1p functions

with:

7.12.6.9 The log1p and logpl functions

In 7.12.6.9#1, append to the Synopsis:

\begin{verbatim}
double log1p(double x);
float log1pf(float x);
long double log1pl(long double x);
_FloatN log1fN(_FloatN x);
_FloatN x log1fN(_FloatN x);
_DecimalN log1dN(_DecimalN x);
_DecimalN x log1dN(_DecimalN x);
\end{verbatim}

In 7.12.6.9#2, replace the first sentence:

The log1p functions compute the base-e (natural) logarithm of 1 plus the argument.

with:

The log1p functions are equivalent to the logpl functions. These functions compute the base-e (natural) logarithm of 1 plus the argument.

Replace 7.12.6.9#3:

[3] The log1p functions return \(\log_e (1 + x)\).

with:

[3] These functions return \(\log_e (1 + x)\).

In F.10.3.9, replace the subclause title:

F.10.3.9 The log1p functions

with:

F.10.3.9 The log1p and logpl functions
After 7.12.6.13, insert the following:

7.12.6.14 The exp2m1 functions

Synopsis

[1] #include <math.h>
   double exp2m1(double x);
   float exp2m1f(float x);
   long double exp2m1l(long double x);
   _FloatN exp2m1fN(_FloatN x);
   _FloatNx exp2m1fNx(_FloatNx x);
   _DecimalN exp2m1dN(_DecimalN x);
   _DecimalNx exp2m1dNx(_DecimalNx x);

Description

[2] The exp2m1 functions compute the base-2 exponential of the argument, minus 1. A range error occurs if finite x is too large or if the magnitude of nonzero x is too small.

Returns

[3] The exp2m1 functions return $2^x - 1$.

7.12.6.15 The exp10 functions

Synopsis

[1] #include <math.h>
   double exp10(double x);
   float exp10f(float x);
   long double exp10l(long double x);
   _FloatN exp10fN(_FloatN x);
   _FloatNx exp10fNx(_FloatNx x);
   _DecimalN exp10dN(_DecimalN x);
   _DecimalNx exp10dNx(_DecimalNx x);

Description

[2] The exp10 functions compute the base-10 exponential of the argument. A range error occurs if the magnitude of finite x is too large.

Returns

7.12.6.16 The \texttt{exp10m1} functions

Synopsis

[1] \verbatim
#include <math.h>
  double \texttt{exp10m1}(double \texttt{x});
  float \texttt{exp10m1f}(float \texttt{x});
  long double \texttt{exp10m1l}(long double \texttt{x});
  \_FloatN \texttt{exp10m1fN}(\_FloatN \texttt{x});
  \_FloatNx \texttt{exp10m1fNx}(\_FloatNx \texttt{x});
  \_DecimalN \texttt{exp10m1dN}(\_DecimalN \texttt{x});
  \_DecimalNx \texttt{exp10m1dNx}(\_DecimalNx \texttt{x});
\verbatim

Description

[2] The \texttt{exp10m1} functions compute the base-10 exponential of the argument, minus 1. A range error occurs if finite \texttt{x} is too large.

Returns

[3] The \texttt{exp10m1} functions return $10^x - 1$.

7.12.6.17 The \texttt{log2p1} functions

Synopsis

[1] \verbatim
#include <math.h>
  double \texttt{log2p1}(double \texttt{x});
  float \texttt{log2p1f}(float \texttt{x});
  long double \texttt{log2p1l}(long double \texttt{x});
  \_FloatN \texttt{log2p1fN}(\_FloatN \texttt{x});
  \_FloatNx \texttt{log2p1fNx}(\_FloatNx \texttt{x});
  \_DecimalN \texttt{log2p1dN}(\_DecimalN \texttt{x});
  \_DecimalNx \texttt{log2p1dNx}(\_DecimalNx \texttt{x});
\verbatim

Description

[2] The \texttt{log2p1} functions compute the base-2 logarithm of 1 plus the argument. A domain error occurs if the argument is less than $-1$. A pole error may occur if the argument equals $-1$.

Returns

[3] The \texttt{log2p1} functions return $\log_2(1 + x)$.

7.12.6.18 The \texttt{log10p1} functions

Synopsis

[1] \verbatim
#include <math.h>
  double \texttt{log10p1}(double \texttt{x});
  float \texttt{log10p1f}(float \texttt{x});
  long double \texttt{log10p1l}(long double \texttt{x});
  \_FloatN \texttt{log10p1fN}(\_FloatN \texttt{x});
  \_FloatNx \texttt{log10p1fNx}(\_FloatNx \texttt{x});
  \_DecimalN \texttt{log10p1dN}(\_DecimalN \texttt{x});
  \_DecimalNx \texttt{log10p1dNx}(\_DecimalNx \texttt{x});
\verbatim
Description

[2] The log10p1 functions compute the base-10 logarithm of 1 plus the argument. A domain error occurs if the argument is less than −1. A pole error may occur if the argument equals −1. A range error occurs if the magnitude of nonzero x is too small.

Returns


After 7.12.7.5, insert the following:

7.12.7.6 The rsqrt functions

Synopsis

[1] #include <math.h>
    double rsqrt(double x);
    float rsqrtf(float x);
    long double rsqrtl(long double x);
    _FloatN rsqrt(N(_FloatN x));
    _FloatNx rsqrtfNx(_FloatNx x);
    _DecimalN rsqrtdN(_DecimalN x);
    _DecimalNx rsqrtdNx(_DecimalNx x);

Description

[2] The rsqrt functions compute the reciprocal of the square root of the argument. A domain error occurs if the argument is less than zero. A pole error may occur if the argument equals zero.

Returns

[3] The rsqrt functions return 1 / \sqrt{x}.

7.12.7.7 The compoundn functions

Synopsis

[1] #include <math.h>
#include <stdint.h>
    double compoundn(double x, intmax_t n);
    float compoundnf(float x, intmax_t n);
    long double compoundnl(long double x, intmax_t n);
    _FloatN compoundn(N(_FloatN x, intmax_t n));
    _FloatNx compoundnfNx(_FloatNx x, intmax_t n);
    _DecimalN compoundndN(_DecimalN x, intmax_t n);
    _DecimalNx compoundndNx(_DecimalNx x, intmax_t n);

Description

[2] The compoundn functions compute 1 plus x, raised to the power n. A domain error occurs if x < −1. A range error may occur if n is too large, depending on x. A pole error may occur if x equals −1 and n < 0.

Returns

7.12.7.8 The rootn functions

Synopsis

[1] #include <math.h>
    #include <stdint.h>

double rootn(double x, intmax_t n);
float rootnf(float x, intmax_t n);
long double rootnl(long double x, intmax_t n);
_FloatN rootnFN(_FloatN x, intmax_t n);
_FloatNx rootn Nx(_FloatNx x, intmax_t n);
_DecimalN rootnNx(_DecimalN x, intmax_t n);

Description

[2] The rootn functions compute the principal nth root of x. A domain error occurs if n is 0 or if x < 0 and n is even. A range error may occur if n is −1. A pole error may occur if x equals zero and n < 0.

Returns

[3] The rootn functions return \( x^{1/n} \).

7.12.7.9 The pown functions

Synopsis

[1] #include <math.h>
    #include <stdint.h>

double pown(double x, intmax_t n);
float pownf(float x, intmax_t n);
long double pownl(long double x, intmax_t n);
_FloatN pownFN(_FloatN x, intmax_t n);
_FloatNx pown Nx(_FloatNx x, intmax_t n);
_DecimalN powndN(_DecimalN x, intmax_t n);
_DecimalNx pownd Nx(_DecimalNx x, intmax_t n);

Description

[2] The pown functions compute x raised to the nth power. A range error may occur. A pole error may occur if x equals zero and n < 0.

Returns

[3] The pown functions return \( x^n \).

7.12.7.10 The powr functions

Synopsis

[1] #include <math.h>

double powr(double x, double y);
float powrf(float x, float y);
long double powrl(long double x, long double y);
_FloatN powrfN(_FloatN x, _FloatN y);
_FloatNx powrf Nx(_FloatNx x, _FloatNx y);
_DecimalN powrdN(_DecimalN x, _DecimalN y);
_DecimalNx powrd Nx(_DecimalNx x, _DecimalNx y);
Description

[2] The \texttt{powr} functions compute $x$ raised to the power $y$ as $\exp(y \times \log(x))$. A domain error occurs if $x < 0$ or if $x$ and $y$ are both zero. A range error may occur. A pole error may occur if $x$ equals zero and finite $y < 0$.

Returns

[3] The \texttt{powr} functions return $x^y$.

After 7.31.6, insert:

7.31.6a Mathematics \texttt{<math.h>}

With the condition that the macro \_\_STDC\_IEC\_60559\_FUNCS\_ is defined, the function names

\begin{verbatim}
crexp crrsqrt cracospi  ccrexpml1 crccompoundn cratanpi  ccexp2 crrootn cratan2pi  ccrexp2ml crrpow crasin
ccexp10 crrcos crasinh
ccexp10ml crrsqrt cratan
ccrllog cssin cratan2
ccrllog2 ccrcos cssinh
ccrllog10 ccrsin crctanh
ccrlg1p cssinpi crctanh
ccrlgpl ccrcospi crcsinh
ccrlg2p1 crtanpi crcosh
ccrlg10p1 crasinpi crctanh
ccrhypot
\end{verbatim}

and the same names suffixed with \texttt{f}, \texttt{l}, \texttt{fN}, \texttt{fNx}, \texttt{dN}, or \texttt{dNx} may be added to the \texttt{<math.h>} header.

In 7.31.6a, attach a footnote to the wording:

With the condition that the macro \_\_STDC\_IEC\_60559\_FUNCS\_ is defined, the function names

where the footnote is:

*) The \texttt{cr} prefix is intended to indicate a correctly rounded version of the function.

After F.10\#2, insert:

[2a] For each single-argument function $f$ in \texttt{<math.h>} whose mathematical counterpart is symmetric (even), $f(x)$ is $f(-x)$ for all rounding modes and for all $x$ in the (valid) domain of the function. For each single-argument function $f$ in \texttt{<math.h>} whose mathematical counterpart is antisymmetric (odd), $f(-x)$ is $-f(x)$ for the IEC 60559 rounding modes roundTiesToEven, roundTiesToAway, and roundTowardZero, and for all $x$ in the (valid) domain of the function. The \texttt{atan2} and \texttt{atan2pi} functions are odd in their first argument.

After F.10.1.7, insert the following:

F.10.1.8 The \texttt{acospi} functions

- \texttt{acospi}(+1) returns +0.
- \texttt{acospi}(x) returns a NaN and raises the "invalid" floating-point exception for $|x| > 1$. 
F.10.1.9 The asinpi functions

- asinpi(±0) returns ±0.
- asinpi(x) returns a NaN and raises the “invalid” floating-point exception for |x| > 1.

F.10.1.10 The atanpi functions

- atanpi(±0) returns ±0.
- atanpi(±∞) returns 1/2.

F.10.1.11 The atan2pi functions

- atan2pi(±0, −0) returns ±1.
- atan2pi(±0, +0) returns ±0.
- atan2pi(±0, x) returns ±1 for x < 0.
- atan2pi(±0, x) returns ±0 for x > 0.
- atan2pi(y, ±0) returns −1/2 for y < 0.
- atan2pi(y, ±0) returns +1/2 for y > 0.
- atan2pi(±y, −∞) returns ±1 for finite y > 0.
- atan2pi(±y, +∞) returns ±0 for finite y > 0.
- atan2pi(±∞, x) returns ±1/2 for finite x.
- atan2pi(±∞, −0) returns ±3/4 for finite x.
- atan2pi(±∞, +0) returns ±1/4 for finite x.

F.10.1.12 The cospi functions

- cospi(±0) returns 1.
- cospi(n + 1/2) returns +0, for integers n.
- cospi(±∞) returns a NaN and raises the “invalid” floating-point exception.

F.10.1.13 The sinpi functions

- sinpi(±0) returns ±0.
- sinpi(±n) returns ±0, for positive integers n.
- sinpi(±∞) returns a NaN and raises the “invalid” floating-point exception.

F.10.1.14 The tanpi functions

- tanpi(±0) returns ±0.
- tanpi(n) returns +0, for positive even and negative odd integers n.
- tanpi(n) returns −0, for positive odd and negative even integers n.
- tanpi(n + 1/2) returns +∞ and raises the “divide-by-zero” floating-point exception, for even integers n.
- tanpi(n + 1/2) returns −∞ and raises the “divide-by-zero” floating-point exception, for odd integers n.
- tanpi(±∞) returns a NaN and raises the “invalid” floating-point exception.
After F.10.3.13, insert the following:

F.10.3.14 The \texttt{exp2m1} functions

- \texttt{exp2m1}(\pm 0) \text{ returns } \pm 0.
- \texttt{exp2m1}(\pm \infty) \text{ returns } -1.
- \texttt{exp2m1}(+\infty) \text{ returns } +\infty.

F.10.3.15 The \texttt{exp10} functions

- \texttt{exp10}(\pm 0) \text{ returns } 1.
- \texttt{exp10}(\pm \infty) \text{ returns } +1.
- \texttt{exp10}(+\infty) \text{ returns } +\infty.

F.10.3.16 The \texttt{exp10m1} functions

- \texttt{exp10m1}(\pm 0) \text{ returns } \pm 0.
- \texttt{exp10m1}(\pm \infty) \text{ returns } -1.
- \texttt{exp10m1}(+\infty) \text{ returns } +\infty.

F.10.3.17 The \texttt{log2p1} functions

- \texttt{log2p1}(\pm 0) \text{ returns } \pm 0.
- \texttt{log2p1}(-1) \text{ returns } -\infty \text{ and raises the “divide-by-zero” floating-point exception.}
- \texttt{log2p1}(x) \text{ returns a NaN and raises the “invalid” floating-point exception for } x < -1.
- \texttt{log2p1}(+\infty) \text{ returns } +\infty.

F.10.3.18 The \texttt{log10p1} functions

- \texttt{log10p1}(\pm 0) \text{ returns } \pm 0.
- \texttt{log10p1}(-1) \text{ returns } -\infty \text{ and raises the “divide-by-zero” floating-point exception.}
- \texttt{log10p1}(x) \text{ returns a NaN and raises the “invalid” floating-point exception for } x < -1.
- \texttt{log10p1}(+\infty) \text{ returns } +\infty.

After F.10.4.5, insert the following:

F.10.4.6 The \texttt{rsqrt} functions

- \texttt{rsqrt}(\pm 0) \text{ returns } +\infty \text{ and raises the “divide-by-zero” floating-point exception.}
- \texttt{rsqrt}(x) \text{ returns a NaN and raises the “invalid” floating-point exception for } x < 0.
- \texttt{rsqrt}(+\infty) \text{ returns } +0.

F.10.4.7 The \texttt{compoundn} functions

- \texttt{compoundn}(x, 0) \text{ returns } 1 \text{ for } x \geq -1.
- \texttt{compoundn}(x, n) \text{ returns a NaN and raises the “invalid” floating-point exception for } x < -1.
- \texttt{compoundn}(+\infty, 0) \text{ returns } 1.
- \texttt{compoundn}(x, 0) \text{ returns } 1 \text{ for } x \text{ a NaN.}
- \texttt{compoundn}(-1, n) \text{ returns } +\infty \text{ and raises the divide-by-zero floating-point exception for } n < 0.
- \texttt{compoundn}(-1, n) \text{ returns } +0 \text{ for } n > 0.
F.10.4.8 The rootn functions

- \texttt{rootn}(\pm0, n) returns \pm\infty and raises the “divide-by-zero” floating-point exception for odd \(n < 0\).
- \texttt{rootn}(\pm0, n) returns +\infty and raises the “divide-by-zero” floating-point exception for even \(n < 0\).
- \texttt{rootn}(\pm0, n) returns +0 for even \(n > 0\).
- \texttt{rootn}(\pm0, n) returns ±0 for odd \(n > 0\).
- \texttt{rootn}(\pm\infty, n) is equivalent to \texttt{rootn}(\pm0, -n) for \(n \neq 0\), except that the “divide-by-zero” floating-point exception is not raised.
- \texttt{rootn}(x, 0) returns a NaN and raises the “invalid” floating-point exception for all \(x\) (including NaN).
- \texttt{rootn}(x, n) returns a NaN and raises the “invalid” floating-point exception for \(x < 0\) and \(n\) even.

F.10.4.9 The pown functions

- \texttt{pown}(x, 0) returns 1 for all \(x\) not a signaling NaN.
- \texttt{pown}(\pm0, n) returns ±\infty and raises the “divide-by-zero” floating-point exception for odd \(n < 0\).
- \texttt{pown}(\pm0, n) returns +\infty and raises the “divide-by-zero” floating-point exception for even \(n < 0\).
- \texttt{pown}(\pm0, n) returns +0 for even \(n > 0\).
- \texttt{pown}(\pm0, n) returns ±0 for odd \(n > 0\).
- \texttt{pown}(\pm\infty, n) is equivalent to \texttt{pown}(\pm0, -n) for \(n \neq 0\), except that the “divide-by-zero” floating-point exception is not raised.

F.10.4.10 The powr functions

- \texttt{powr}(x, \pm0) returns 1 for finite \(x > 0\).
- \texttt{powr}(\pm0, y) returns +\infty and raises the “divide-by-zero” floating-point exception for finite \(y < 0\).
- \texttt{powr}(\pm0, -\infty) returns +\infty.
- \texttt{powr}(\pm0, y) returns +0 for \(y > 0\).
- \texttt{powr}(\pm1, y) returns 1 for finite \(y\).
- \texttt{powr}(x, y) returns a NaN and raises the “invalid” floating-point exception for \(x < 0\).
- \texttt{powr}(\pm\infty, \pm0) returns a NaN and raises the “invalid” floating-point exception.
- \texttt{powr}(\pm\infty, \pm0) returns a NaN and raises the “invalid” floating-point exception.
- \texttt{powr}(1, \pm\infty) returns a NaN and raises the “invalid” floating-point exception.

8 Reduction functions in \\texttt{<math.h>}

This clause specifies changes to C11 + TS18661-1 + TS18661-2 + TS18661-3 to include functions that support reduction operations recommended by IEC 60559.

Changes to C11 + TS18661-1 + TS18661-2 + TS18661-3:

After 7.12.13a, insert the following:

7.12.13b Reduction functions

The functions in this subclause should be implemented so that intermediate computations do not overflow or underflow.

Functions computing sums of length \(n = 0\) return the value +0. Functions computing products of length \(n = 0\) return the value 1 and store the scale factor 0 in the object pointed to by \texttt{sfptr}.
7.12.13b.1 The \texttt{reduc\_sum} functions

Synopsis

[1] #include <math.h>
#include <stddef.h>
  double reduc\_sum(size\_t n, const double p[static n]);
  float reduc\_sumf(size\_t n, const float p[static n]);
  long double reduc\_suml(size\_t n, const long double p[static n]);
  _FloatN reduc\_sum\_N(size\_t n, const _FloatN p[static n]);
  _FloatNx reduc\_sum\_Nx(size\_t n, const _FloatNx p[static n]);
  _DecimalN reduc\_sum\_D(size\_t n, const _DecimalN p[static n]);
  _DecimalNx reduc\_sum\_Nx(size\_t n, const _DecimalNx p[static n]);

Description

[2] The \texttt{reduc\_sum} functions compute the sum of the \texttt{n} members of array \texttt{p}: \texttt{$\sum_{i=0}^{n-1} p[i]$}. A range error may occur.

Returns

[3] The \texttt{reduc\_sum} functions return the computed sum.

7.12.13b.2 The \texttt{reduc\_sumabs} functions

Synopsis

[1] #include <math.h>
#include <stddef.h>
  double reduc\_sumabs(size\_t n, const double p[static n]);
  float reduc\_sumabsf(size\_t n, const float p[static n]);
  long double reduc\_sumabsl(size\_t n, const long double p[static n]);
  _FloatN reduc\_sumabs\_N(size\_t n, const _FloatN p[static n]);
  _FloatNx reduc\_sumabs\_Nx(size\_t n, const _FloatNx p[static n]);
  _DecimalN reduc\_sumabs\_D(size\_t n, const _DecimalN p[static n]);
  _DecimalNx reduc\_sumabs\_Nx(size\_t n, const _DecimalNx p[static n]);

Description

[2] The \texttt{reduc\_sumabs} functions compute the sum of the absolute values of the \texttt{n} members of array \texttt{p}: \texttt{$\sum_{i=0}^{n-1} |p[i]|$}. A range error may occur.

Returns

[3] The \texttt{reduc\_sumabs} functions return the computed sum.
7.12.13b.3 The reduc_sumsq functions

Synopsis

[1] #include <math.h>
#include <stddef.h>
double reduc_sumsq(size_t n, const double p[static n]);
float reduc_sumsqf(size_t n, const float p[static n]);
long double reduc_sumsqI(size_t n, const long double p[static n]);
_FloatN reduc_sumsqfN(size_t n, const _FloatN p[static n]);
_FloatNx reduc_sumsqfNx(size_t n, const _FloatNx p[static n]);
_DecimalN reduc_sumsqfN(size_t n, const _DecimalN p[static n]);
_FloatNx reduc_sumsqfNx(size_t n, const _DecimalN p[static n]);

Description

[2] The reduc_sumsq functions compute the sum of squares of the values of the n members of array p: \( \sum_{i=0}^{n-1} (p[i] \times p[i]) \). A range error may occur.

Returns


7.12.13b.4 The reduc_sumprod functions

Synopsis

[1] #include <math.h>
#include <stddef.h>
double reduc_sumprod(size_t n, const double p[static n],
const double q[static n]);
float reduc_sumprodf(size_t n, const float p[static n],
const float q[static n]);
long double reduc_sumprodl(size_t n, const long double p[static n],
const long double q[static n]);
_FloatN reduc_sumprodfN(size_t n, const _FloatN p[static n],
const _FloatN q[static n]);
_FloatNx reduc_sumprodfNx(size_t n, const _FloatNx p[static n],
const _FloatNx q[static n]);
_DecimalN reduc_sumprodN(size_t n, const _DecimalN p[static n],
const _DecimalN q[static n]);
_FloatNx reduc_sumprodNx(size_t n, const _DecimalN p[static n],
const _DecimalN q[static n]);

Description

[2] The reduc_sumprod functions compute the dot product of the sequences of members of the arrays p and q: \( \sum_{i=0}^{n-1} (p[i] \times q[i]) \). A range error may occur.

Returns

7.12.13b.5 The scaled_prod functions

Synopsis

[1] #include <math.h>
#include <stddef.h>
#include <stdint.h>

double scaled_prod(size_t n, const double p[static restrict n],
                   intmax_t * restrict sfptr);
float scaled_prodf(size_t n, const float p[static restrict n],
                   intmax_t * restrict sfptr);
long double scaled_prodl(size_t n, const long double p[static restrict n],
                        intmax_t * restrict sfptr);
_FloatN scaled_prodfN(size_t n, const _FloatN p[static restrict n],
                      intmax_t * restrict sfptr);
_FloatNx scaled_prodfNx(size_t n, const _FloatNx p[static restrict n],
                       intmax_t * restrict sfptr);
.DecimalN scaled_prodfN(size_t n, const _DecimalN p[static restrict n],
                       intmax_t * restrict sfptr);
.DecimalNx scaled_prodfNx(size_t n, const _DecimalNx p[static restrict n],
                       intmax_t * restrict sfptr);

Description

[2] The scaled_prod functions compute a scaled product pr of the n members of the array p and a scale factor sf, such that pr × b^sf = Π_{i=0}^{n-1} p[i], where b is the radix of the type. These functions store the scale factor sf in the object pointed to by sfptr. A domain error occurs if the scale factor is outside the range of the intmax_t type. The functions should not cause a range error.

Returns


7.12.13b.6 The scaled_prodsum functions

Synopsis

[1] #include <math.h>
#include <stddef.h>
#include <stdint.h>

double scaled_prodsum(size_t n, const double p[static restrict n],
                      const double q[static restrict n],
                      intmax_t * restrict sfptr);
float scaled_prodsumf(size_t n, const float p[static restrict n],
                      const float q[static restrict n],
                      intmax_t * restrict sfptr);
long double scaled_prodsuml(size_t n, const long double p[static restrict n],
                            const long double q[static restrict n],
                            intmax_t * restrict sfptr);
_FloatN scaled_prodsumfN(size_t n, const _FloatN p[static restrict n],
                        const _FloatN q[static restrict n],
                        intmax_t * restrict sfptr);
_FloatNx scaled_prodsumfNx(size_t n, const _FloatNx p[static restrict n],
                        const _FloatNx q[static restrict n],
                        intmax_t * restrict sfptr);
.DecimalN scaled_prodsumN(size_t n, const _DecimalN p[static restrict n],
                        const _DecimalN q[static restrict n],
                        intmax_t * restrict sfptr);
.DecimalNx scaled_prodsumNx(size_t n, const _DecimalNx p[static restrict n],
                        const _DecimalNx q[static restrict n],
                        intmax_t * restrict sfptr);
Description

[2] The `scaled_prodsnum` functions compute a scaled product `pr` of the sums of the corresponding members of the arrays `p` and `q` and a scale factor `sf`, such that `pr \times b^{sf} = \prod_{i=0}^{n-1}(p[i] + q[i])`, where `b` is the radix of the type. These functions store the scale factor `sf` in the object pointed to by `sfptr`. A domain error occurs if the scale factor is outside the range of the `intmax_t` type. These functions should not cause a range error.

Returns


7.12.13b.7 The `scaled_proddiff` functions

Synopsis

[1] `#include <math.h>
#include <stddef.h>
#include <stdint.h>`

```c
double scaled_proddiff(size_t n, const double p[static restrict n],
                       const double q[static restrict n], intmax_t * restrict sfptr);
float scaled_proddifff(size_t n, const float p[static restrict n],
                       const float q[static restrict n], intmax_t * restrict sfptr);
long double scaled_proddiffll(size_t n,
                               const long double p[static restrict n],
                               const long double q[static restrict n], intmax_t * restrict sfptr);
_FloatN scaled_proddiffN(size_t n,
                         const _FloatN p[static restrict n],
                         const _FloatN q[static restrict n], intmax_t * restrict sfptr);
_FloatNx scaled_proddiffNx(size_t n,
                          const _FloatNx p[static restrict n],
                          const _FloatNx q[static restrict n], intmax_t * restrict sfptr);
_DecimalN scaled_proddiffD(size_t n,
                          const _DecimalN p[static restrict n],
                          const _DecimalN q[static restrict n], intmax_t * restrict sfptr);
_DecimalNx scaled_proddiffDx(size_t n,
                            const _DecimalNx p[static restrict n],
                            const _DecimalNx q[static restrict n], intmax_t * restrict sfptr);
```

Description

[2] The `scaled_proddiff` functions compute a scaled product `pr` of the differences of the corresponding members of the arrays `p` and `q` and a scale factor `sf`, such that `pr \times b^{sf} = \prod_{i=0}^{n-1}(p[i] - q[i])`, where `b` is the radix of the type. These functions store the scale factor `sf` in the object pointed to by `sfptr`. A domain error occurs if the scale factor is outside the range of the `intmax_t` type. These functions should not cause a range error.

Returns


After F.10.10a, insert

F.10.10b Reduction functions

The functions in this subclause return a NaN if any member of an array argument is a NaN, unless explicitly specified otherwise.
The `reduc_sum`, `reduc_sumabs`, `reduc_sumsq`, and `reduc_sumprod` functions avoid overflow and underflow in intermediate computation. They raise the “overflow” or “underflow” floating-point exception if and only if the determination of the final result overflows or underflows.

The `scaled_prod`, `scaled_prodsum`, and `scaled_proddiff` functions do not raise the “overflow” or “underflow” floating-point exceptions.

The functions in this subclause do not raise the “divide-by-zero” floating-point exception.

F.10.10.b.1 The `reduc_sum` functions

- `reduc_sum(n, p)` returns a NaN if any member of array `p` is a NaN.
- `reduc_sum(n, p)` returns a NaN and raises the “invalid” floating-point exception if any two members of array `p` are infinities with different signs.
- Otherwise, `reduc_sum(n, p)` returns $\pm \infty$ if the members of `p` include one or more infinities $\pm \infty$ (with the same sign).

F.10.10.b.2 The `reduc_sumabs` functions

- `reduc_sumabs(n, p)` returns $+\infty$ if any member of array `p` is an infinity.
- Otherwise, `reduc_sumabs(n, p)` returns a NaN if any member of array `p` is a NaN.

F.10.10.b.3 The `reduc_sumsq` functions

- `reduc_sumsq(n, p)` returns $+\infty$ if any member of array `p` is an infinity.
- Otherwise, `reduc_sumsq(n, p)` returns a NaN if any member of array `p` is a NaN.

F.10.10.b.4 The `reduc_sumprod` functions

- `reduc_sumprod(n, p, q)` returns a NaN if any member of array `p` or `q` is a NaN.
- `reduc_sumprod(n, p, q)` returns a NaN and raises the “invalid” floating-point exception if any of the products has a zero and an infinite factor.
- `reduc_sumprod(n, p, q)` returns a NaN and raises the “invalid” floating-point exception if any two of the products are (exact) infinities with different signs.
- Otherwise, `reduc_sumprod(n, p, q)` returns $\pm \infty$ if one or more of the products are (exactly) $\pm \infty$ (with the same sign).

F.10.10.b.5 The `scaled_prod` functions

- `scaled_prod(n, p, sfptr)` returns a NaN if any member of array `p` is a NaN.
- `scaled_prod(n, p, sfptr)` returns a NaN and raises the “invalid” floating-point exception if any two members of array `p` are a zero and an infinity.
- Otherwise, `scaled_prod(n, p, sfptr)` returns an infinity if any member of array `p` is an infinity.
- Otherwise, `scaled_prod(n, p, sfptr)` returns a zero if any member of array `p` is a zero.
- Otherwise, `scaled_prod(n, p, sfptr)` returns a NaN and raises the “invalid” floating-point exception if the scale factor is outside the range of the `intmax_t` type.
F.10.10b.6 The scaled_prodsum functions

- scaled_prodsum(n, p, q, sfptr) returns a NaN if any member of p or q is a NaN.
- scaled_prodsum(n, p, q, sfptr) returns a NaN and raises the “invalid” floating-point exception if any two factors (each of which is a sum) are zero and infinity (exactly).
- scaled_prodsum(n, p, q, sfptr) returns a NaN and raises the “invalid” floating-point exception if any of the sums is of two infinities with different signs.
- Otherwise, scaled_prodsum(n, p, q, sfptr) returns an infinity if any factor is an exact infinity.
- Otherwise, scaled_prodsum(n, p, q, sfptr) returns a zero if any factor is a zero.
- Otherwise, scaled_prodsum(n, p, q, sfptr) returns a NaN and raises the “invalid” floating-point exception if the scale factor is outside the range of the intmax_t type.

F.10.10b.7 The scaled_proddiff functions

- scaled_proddiff(n, p, q, sfptr) returns a NaN if any member of p or q is a NaN.
- scaled_proddiff(n, p, q, sfptr) returns a NaN and raises the “invalid” floating-point exception if any two factors (each of which is a difference) are zero and infinity (exactly).
- scaled_proddiff(n, p, q, sfptr) returns a NaN and raises the "invalid" floating-point exception if any of the differences is of two infinities with the same signs.
- Otherwise, scaled_proddiff(n, p, q, sfptr) returns an infinity if any factor is an exact infinity.
- Otherwise, scaled_proddiff(n, p, q, sfptr) returns a zero if any factor is a zero.
- Otherwise, scaled_proddiff(n, p, q, sfptr) returns a NaN and raises the “invalid” floating-point exception if the scale factor is outside the range of the intmax_t type.

9 Future directions for <complex.h>

This clause extends the list of function names reserved for future library directions under <complex.h> to include complex versions of math functions that this Part of Technical Specification 18661 adds to C11.

Change to C11 + TS18661-1 + TS18661-2 + TS18661-3:

In 7.31.1, add the following after the list of function names:

and, with the condition that the macro __STDC_IEC_60559_FUNCS__ is defined, the functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Function</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>cexp2ml</td>
<td>crsqrt</td>
<td>casinpi</td>
</tr>
<tr>
<td>cexp10</td>
<td>ccompoundn</td>
<td>catanpi</td>
</tr>
<tr>
<td>cexp10m1</td>
<td>crootn</td>
<td>ccospo</td>
</tr>
<tr>
<td>clogp1</td>
<td>cpown</td>
<td>csinpi</td>
</tr>
<tr>
<td>clog2p1</td>
<td>cpowr</td>
<td>ctanpi</td>
</tr>
<tr>
<td>clog10p1</td>
<td>cacospo</td>
<td></td>
</tr>
</tbody>
</table>

35 10 Type-generic macros <tgmath.h>

The following changes to C11 + TS18661-1 + TS18661-2 + TS18661-3 enhance the specification of type-generic macros in <tgmath.h> to apply to math functions that this Part of Technical Specification 18661 adds to C11.

Changes to C11 + TS18661-1 + TS18661-2 + TS18661-3:

In 7.25#5, change:

For each unsuffixed function in <math.h> without a c-prefixed counterpart in <complex.h> (except modf, setpayload, setpayloadsig, and canonicalize) …
For each unsuffixed function in `<math.h>` without a `c`-prefixed counterpart in `<complex.h>` (except `modf`, `setpayload`, `setpayloadsig`, `canonicalize`, and the reduction functions in 7.12.13b)...

In 7.25#5, add the following to the list of type-generic macros:

<table>
<thead>
<tr>
<th></th>
<th>exp2m1</th>
<th>rsqrt</th>
<th>asinpi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>exp10</td>
<td>compoundn</td>
<td>atanpi</td>
</tr>
<tr>
<td></td>
<td>exp10m1</td>
<td>rootn</td>
<td>atan2pi</td>
</tr>
<tr>
<td></td>
<td>logp1</td>
<td>pown</td>
<td>cospi</td>
</tr>
<tr>
<td></td>
<td>log2p1</td>
<td>powr</td>
<td>sinpi</td>
</tr>
<tr>
<td>10</td>
<td>log10p1</td>
<td>acospi</td>
<td>tanpi</td>
</tr>
</tbody>
</table>
Bibliography


